Shortest Path Problem

(directed)

Input: $G = (V, E)$, $w: E \to \mathbb{R}$, $s, t \in V$ $\quad (|V| = n, |E| = m)$

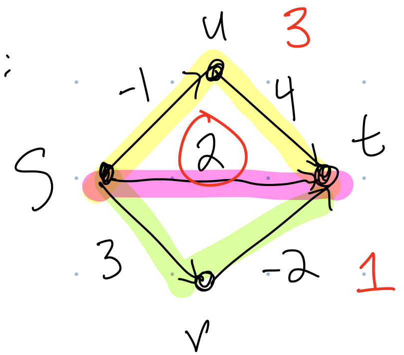
Output: Path from s to t in G

ex: $P = ((s,u), (u,v), \ldots (r,t))$

s.t. $L(P) = \sum_{e \in P} w(e)$ is minimized

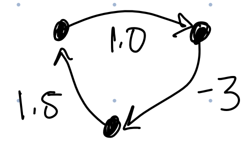
Example:

G:



Shortest Path: $((s,v),(v,t))$

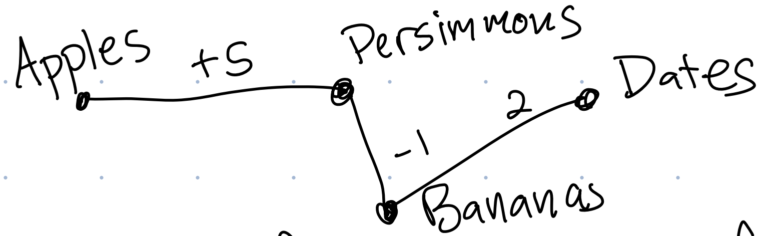
Negative Cycle:



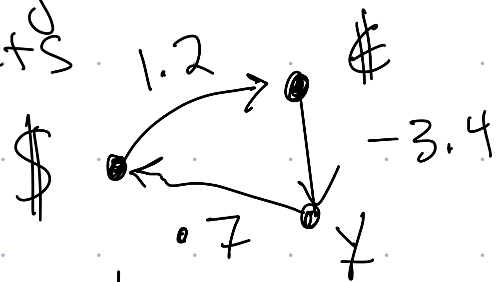
Various Approaches

- Greedy → Dijkstra's → edges have positive weight
- Brute Force → BFS → edges all have weight 1.
- Dynamic → Bellman-Ford → edge have neg weight
  → fails if path must avoid neg cycles → NP-Hard

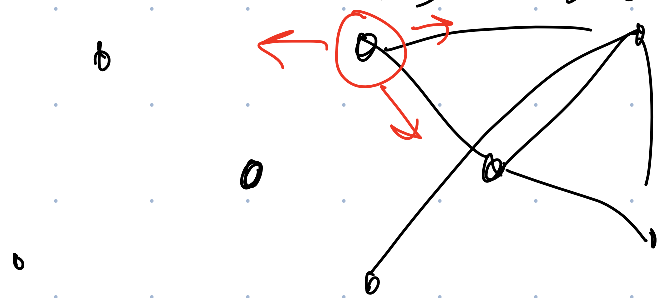
Applications: Bartering



Arbitrage: Finding inefficiencies in financial markets



Data Routing in Networks - Distributed Graph



Cold Weather Plans?

Bartering	Stakeholders	Well-Being	Autonomy	Justice
	Merchants	taken advantage of, get customers	Won't even know interacting with alg	unjust
	Customer | Buyers	$ $		Wealthy might have more success → more $ → access to algorithm

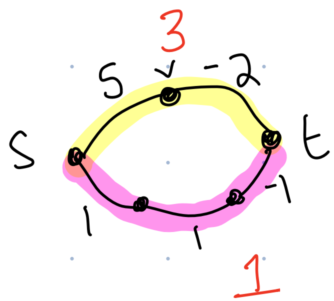
Arbitrage				
	Traders	freeish $		Richer traders have access to better servers, location
	Environment			location of data centers

Defining Subproblems

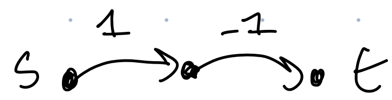
Clever idea: $P_{u,i}$ = shortest path from s to u that uses at most i edges.

What is $L(P_{t,1})$, $L(P_{t,2})$, $L(P_{t,3})$ for the following graph

$L(P_{t,2}) = 3$



$P_{t,2} = ((s,v),(v,t))$



no path, no edge

$\boxed{\text{A) } \infty,\ 3,\ 1}$ B) ∞, 2, 3 C) 0, 2, 3 D) 0, 3, 1

Last choice a strategy makes? Last vertex prior to t.

Designing a Dynamic Prog. Alg.

- Recurrence relation for $P_{u,i}$ = shortest path from s to u that uses at most i edges

$P_{u,i} = \begin{cases} \underline{\qquad\qquad} & \text{if shortest path with } i \text{ edges goes through } v \text{ directly prior to } u \\ \underline{\qquad\qquad} & \text{if shortest path with } i \text{ edges goes through } w \text{ directly prior to } u \\ \vdots & \\ \underline{\qquad\qquad} & \text{if shortest path to } u \text{ uses less than } i \text{ edges} \end{cases}$

- Recurrence relation for objective function $\left(\begin{array}{l} w(u,u) = 0 \\ w(u,v) = \infty \text{ if no edge} \end{array} \right)$

$L(P_{u,i}) = \begin{cases} \underline{\qquad\qquad\qquad\qquad\qquad\qquad} \\ \text{Base case: } \underline{\qquad\qquad\qquad\qquad\qquad\qquad} \end{cases}$

Helpful notation $\rightarrow \min_{u \in S} \{f(u)\}$