CS333 - Problem Set 4

1. Consider the 2-qubit state $|\psi\rangle = \frac{1}{\sqrt{2}}|01\rangle_{AB} - \frac{1}{\sqrt{2}}|10\rangle_{AB}$. This state has some strange properties - in particular, as you'll show in this problem, the two qubits are perfectly anticorrelated. It's behavior is so strange, it caused Einstein to believe that quantum mechanics couldn't possibly completely describe reality.

   (a) Suppose Alice and Bob each measure their qubit in the same basis. That is, Alice and Bob both each apply the measurement $M(\eta,\chi) = \{|\phi_0(\eta,\chi)\rangle, |\phi_1(\eta,\chi)\rangle\}$, where $|\phi_0(\eta,\chi)\rangle = \cos\eta|0\rangle + e^{i\chi}\sin\eta|1\rangle$ and $|\phi_1(\eta,\chi)\rangle = -\sin\eta|0\rangle + e^{i\chi}\cos\eta|1\rangle$. Show that if Alice gets outcome $|\phi_0(\eta,\chi)\rangle$, Bob will get outcome $|\phi_1(\eta,\chi)\rangle$, while if Alice gets outcome $|\phi_1(\eta,\chi)\rangle$, Bob will get outcome $|\phi_0(\eta,\chi)\rangle$. $M(\eta,\chi)$ is "generic" in that we can make $M(\eta,\chi)$ represent any possible single qubit measurement by our choice of χ and η. (η is "eta," χ is "chi.")

   (b) What is the overall probability that Alice gets outcome $|\phi_0(\eta,\chi)\rangle$? What is the probability that she gets $|\phi_1(\eta,\chi)\rangle$?

   (c) This anticorrelation is strange because of the following thought experiment. Suppose Alice took her qubit to the moon, and Bob stayed on Earth. Now Alice performs her measurement first, and suppose she gets outcome $|\phi_0(\eta,\chi)\rangle$. Then, even if Bob performs his measurement before any lightspeed communication can have happened between Alice and Bob, Bob's qubit somehow knows to choose the opposite outcome. This will happen no matter which values of η and χ Alice and Bob choose. People thought that this might enable faster-than-light communication, but explain why Part (b) of this question rules out faster-than-light communication.

   (d) We also can have classical (non-quantum) bits that are probabilistically anti-correlated. Consider the following situation. I put a red sock in one box and a blue sock in another box, and I give one box to Alice and one box to Bob, without telling them which box contains which sock. Once Alice opens her box and sees a blue sock, she immediately knows that Bob's box contains a red sock. Why is the quantum situation stranger than this classical situation?

2. Alice and Bob are playing a game where the referee sends them each a qubit that is part of a 2-qubit state. The referee promises that the 2-qubit state is one of two options: $|\psi_0\rangle_{AB}$ or $|\psi_1\rangle_{AB}$, but doesn't tell them which. Alice and Bob's goal is to decide which state they were given. In order to do this, they can each make a local measurement on their part of the state (i.e. they can make a quantum measurement on their individual qubit). After the measurement, they can communicate classically (for example, have a conversation on the phone), discuss their outcomes, and try to decide if the state is $|\psi_0\rangle_{AB}$ or $|\psi_1\rangle_{AB}$. For each of the following games, either give a strategy such that Alice and Bob can always win, or explain why they will always lose with some probability.

(a)

$$
\begin{aligned}
|\psi_0\rangle_{AB} &= \frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB}) \\
|\psi_1\rangle_{AB} &= \frac{1}{\sqrt{2}}(|01\rangle_{AB} + |10\rangle_{AB})
\end{aligned} \tag{1}
$$

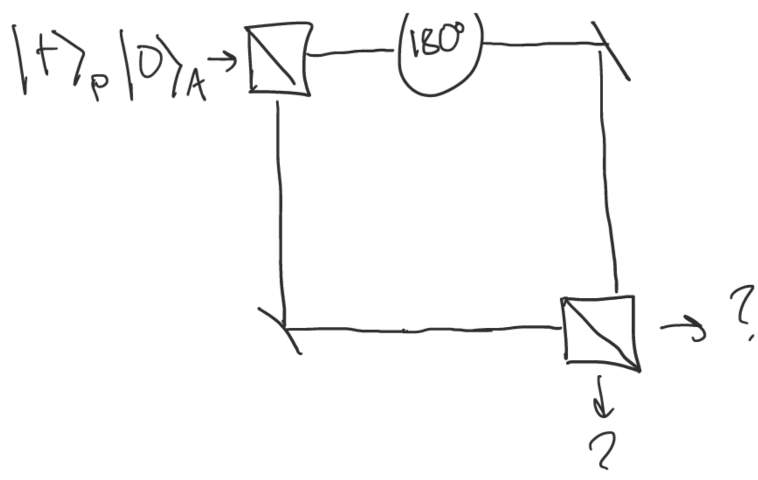
(b)

$$
\begin{aligned}
|\psi_0\rangle_{AB} &= \frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB}) \\
|\psi_1\rangle_{AB} &= |00\rangle_{AB}
\end{aligned} \tag{2}
$$

(c)

$$
\begin{aligned}
|\psi_0\rangle_{AB} &= \frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB}) \\
|\psi_1\rangle_{AB} &= \frac{1}{\sqrt{2}}(|00\rangle_{AB} - |11\rangle_{AB})
\end{aligned} \tag{3}
$$

3. **[Moved to next week's pset]**



Consider the interferometer in the figure above.

(a) What will be the polarization of the photon that exits the interferometer, and will it exit downwards or rightwards (or both)? Note that a 180° lens turns $|0\rangle \to -|0\rangle$

(b) In Problem Set 2, no. 6, you showed that if you multiply a state by -1, it is not physically different from a state that is not multiplied by -1. (Note that $-1 = e^{i\pi}$.) However, in part (a) of this problem, we see that multiplying by -1 does change the physical outcome relative to the case with no lens in the interferometer. (What happens with no lens?) What is different here from PS3 no. 4?

4. Pick either (a) or (b) (or both!) to answer. (a) is likely of more interest to those who like physics, and (b) is likely of more interest to those who like computer science.

(a) Experimental demonstrations of the CHSH game are used not just to show off how cool quantum mechanics is, but also to prove that quantum mechanics is the true theory that describes reality. If you can win the CHSH game with high enough probability, it rules out alternative theories of reality. (For example, "hidden variable theories" that claim that when qubits are entangled, they somehow decide ahead of time which outcomes will occur when some measurement is made in the future.) However, if the CHSH experiments are not performed in just the right way, there are "loopholes" that would allow the results to be explained by factors other than quantum mechanics. Read these two articles Good Bell Test, Bell Test with Starlight, and answer the questions below. (Bell Test is another name for the CHSH Game. The "test" is whether you can win with high enough probability. Note, in the physical implementation of most of these tests, Alice and Bob try to win using a pair of entangled photons that are created at the mid-point between their locations, and then one half of the entangled photon is sent to each of Alice and Bob. When they talk about "measurement settings" they are referring to the questions the referee sends to each of Alice and Bob.):

  i. Describe the three loopholes in your own words, and describe why each loophole could allow some other factor (other than quantum mechanics) to explain why Alice and Bob win with higher than expected probability, and what experimentalists try to do to over come each loophole.

  ii. Please explain why the Detection and Locality Loopholes tend to be hard to close at the same time.

(b) Of of the most important building blocks for creating secure cryptography is the ability to create truly random numbers. For example, our everyday computers don't generate truly random numbers. Instead, when you generate a random number using e.g. python or java, the program takes a seed number, like the exact time, and applies a complicated function to turn the seed into number that looks random. However, if an adversary knew about what time you generated your random number, they would have some idea of the seed and hence a pretty good guess as to what the random number is. If you were trying to use that random number to create a secret key, the adversary might then be able to learn your message. Or maybe your adversary has hidden a program inside your computer which is called every time you try to generate a random number, and which generates a number that looks random to you, but which is actually a number of the adversaries choosing. Then your secret message would be really insecure.

  i. For applications where your randomness has to be really random, but not necessarily secret, one way around this problem is to use noise generated by nature. For example, how is randomness generated at Random.org? If you are extremely paranoid about whether your numbers are truly random, versus being chosen by some adversary, why might this not convince you?

  ii. Read this American Scientist piece by Scott Aaronson on generating true randomness from the CHSH game. (You can skip the sections "Refuting Determinism" and "Infinite Expansion" if you want.)

    A. Explain why it is so important that the referee's questions be random in order to generate randomness from the CHSH game. (How could the players cheat to create numbers that look random but which are really not?)

B. In your own words, (and using our notation from class rather than from the article) explain the basic idea of how to do randomness expansion with the CHSH game to do randomness expansion.