S.KIMMEL

Strategy

state to send $= a|0\rangle + b|1\rangle$

ebit shared

1. A & B start with $|\psi\rangle_{A_1} |\beta_{00}\rangle_{A_2 B}$ (1 ebit)
2. Alice measures A_1 and A_2 (this destroys entanglement) using Bell Basis
3. Alice sends outcome of measurement to Bob (2 cbits)
4. Bob applies a unitary to his system B based on Alice's cbits

What state does Bob end up with in each case?

Which unitary should Bob apply for each outcome?

S.KIMMEL

1. Alice measures qubits A_1 & A_2 in Bell basis (recall strategy from composite systems)

$$|\psi\rangle_{A_1}|\beta_{00}\rangle_{A_2B} = (a|0\rangle+b|1\rangle)\otimes\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$$
$$= \frac{1}{\sqrt{2}}\left(a|000\rangle + a|011\rangle + b|100\rangle + b|111\rangle\right)_{A_1A_2B}$$

$$\begin{aligned}
|\psi\rangle|\beta_{00}\rangle = \\
&+ |\beta_{00}\rangle\langle\beta_{00}|_{A_1A_2}\otimes I_B \ |\psi\rangle|\beta_{00}\rangle &&= \frac{1}{2}|\beta_{00}\rangle(a|0\rangle+b|1\rangle) \quad * \\
&+ |\beta_{01}\rangle\langle\beta_{01}|_{A_1A_2}\otimes I_B \ |\psi\rangle|\beta_{00}\rangle &&+ \frac{1}{2}|\beta_{01}\rangle(a|1\rangle+b|0\rangle) \\
&+ |\beta_{10}\rangle\langle\beta_{10}|_{A_1A_2}\otimes I_B \ |\psi\rangle|\beta_{00}\rangle &&+ \frac{1}{2}|\beta_{10}\rangle(a|0\rangle-b|1\rangle) \\
&+ |\beta_{11}\rangle\langle\beta_{11}|_{A_1A_2}\otimes I_B \ |\psi\rangle|\beta_{00}\rangle &&+ \frac{1}{2}|\beta_{11}\rangle(a|1\rangle-b|0\rangle)
\end{aligned}$$

example of calculation (line $*$)

$$\boxed{\begin{aligned}\langle\beta_{00}|00\rangle &= \frac{1}{\sqrt{2}}\\ \langle\beta_{00}|10\rangle &= 0\\ \langle\beta_{00}|01\rangle &= 0\\ \langle\beta_{00}|11\rangle &= \frac{1}{\sqrt{2}}\end{aligned}}$$

$$|\beta_{00}\rangle\langle\beta_{00}|\otimes I\frac{1}{\sqrt{2}}\left(a|000\rangle + a|011\rangle + b|100\rangle + b|111\rangle\right)$$
$$= \frac{1}{\sqrt{2}}\Big[\langle\beta_{00}|00\rangle a|\beta_{00}\rangle|0\rangle + \langle\beta_{00}|01\rangle a|\beta_{00}\rangle|1\rangle$$
$$+ \langle\beta_{00}|10\rangle b|\beta_{00}\rangle|0\rangle + \langle\beta_{00}|11\rangle b|\beta_{00}\rangle|1\rangle\Big]$$
$$= \frac{1}{\sqrt{2}}\left[\frac{a}{\sqrt{2}}|\beta_{00}\rangle|0\rangle + \frac{b}{\sqrt{2}}|\beta_{00}\rangle|1\rangle\right] = \frac{1}{2}|\beta_{00}\rangle\underbrace{(a|0\rangle+b|1\rangle)}_{\text{properly normalized.}}$$

→ All 4 outcomes occur with equal probability $\left(\frac{1}{4}\right)$

S.KIMMEL

Alice's Outcome	Bob's State B
$\vert\beta_{00}\rangle$	$\vert\psi\rangle = a\vert 0\rangle + b\vert 1\rangle$
$\vert\beta_{01}\rangle$	$X\vert\psi\rangle = b\vert 0\rangle + a\vert 1\rangle$
$\vert\beta_{10}\rangle$	$Z\vert\psi\rangle = a\vert 0\rangle - b\vert 1\rangle$
$\vert\beta_{11}\rangle$	$ZX\vert\psi\rangle = b\vert 0\rangle - a\vert 1\rangle$

2. Alice uses 2 cbits to tell Bob her outcome.
3. Based on this info, Bob applies I, $\underset{X}{X^{-1}}$, $\underset{Z}{Z^{-1}}$, $\underset{ZX}{(ZX)^{-1}}$ to B, recovers the state $|\psi\rangle = a|0\rangle + b|1\rangle$.

Big Picture:

Alice's qubit, which was in state $|\psi\rangle = a|0\rangle + b|1\rangle$, is now the state of Bob's qubit. She never had to communicate a, b. It just shows up in Bob's possession. Pretty crazy!

S.KIMMEL

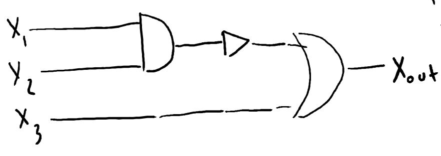
Classical Models of Computation (that helped inspire quantum model)

- Circuit (usual classical model)
- Reversible (b/c unitaries are reversible)
- Probabilistic (b/c measurements are probabilistic)

Circuit - way of describing functions on bits

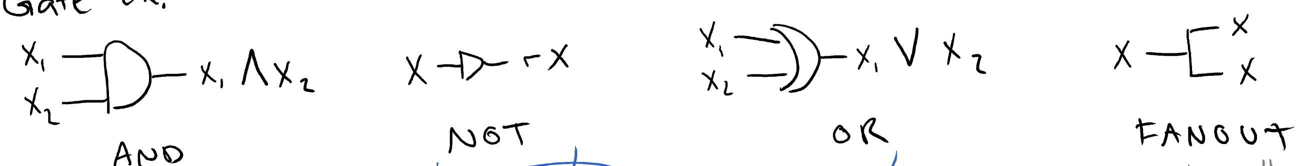
bit: $x \in \{0,1\}$ time →

Set of all 3-bit strings
e.g. $\{0,1\}^2 = \{00, 01, 10, 11\}$ (x_1 x_2)



implements the function $\approx f: \{0,1\}^3 \to \{0,1\}$

Gate ex:



universal universal

Not technically necessary for universality, but useful

Gate set is <u>universal</u>, can compute any $f: \{0,1\}^n \to \{0,1\}^m$

↑ Generalized Boolean function