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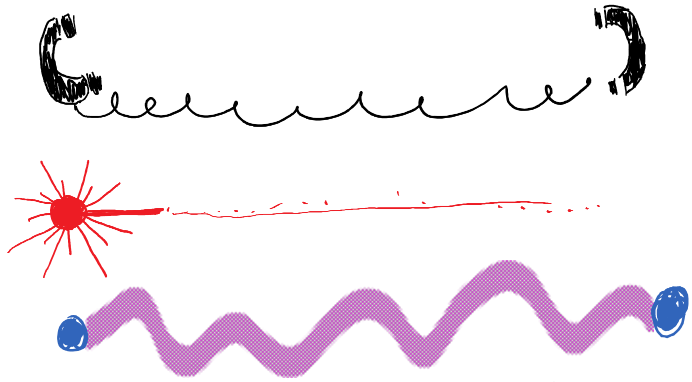
Entanglement & Communication Tasks

Goals: • Be able to describe + analyze superdense coding + teleportation
• Describe properties of entanglement

Announcements: Midterm Details, Timing, Cover, Reflection, • Hints • Discuss why in class, PS 4 Due, Quiz By next class ex: global phase

Alice   Bob

Resources



"cbit" = 1 bit classical info

"qbit" = 1 qubit

"ebit" = 2 qubits in entangled state $|\phi^+\rangle$, Alice and Bob can make measurements or do operations on their system

$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

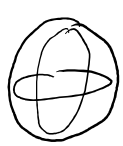
after protocol, entanglement is gone → no longer share b/c either measure, or send

• $R(1 \text{ cbit})$ ≤
• 1 cbit (obvious)
• 1 qbit (A sends $|0\rangle$ or $|1\rangle$, B measures)
• ~~n ebits~~ (can't send info faster than light)

"resources required to send 1 classical bit from Alice to Bob"

• $R(2 \text{ cbits})$ ≤
• 2 cbits
• 2 qbits (Holevo's Thm: can only extract n bits from n qbits)
• 1 ebit + 1 qbit "SUPER DENSE CODING"

Bloch Sphere



Exercise: why doesn't this strategy work

$00 \rightarrow |0\rangle$
$01 \rightarrow |1\rangle$
$10 \rightarrow |+\rangle$
$11 \rightarrow |-\rangle$

Try different measurements

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Strategy

1. Alice & Bob share ebit
2. Alice applies unitary to her state based on what cbits she wants to send to Bob
3. Alice sends her qubit (half of ebit) to Bob

* Entanglement used up (1 ebit)
* 1 qubit sent by quantum channel (1 qbit)

Alice's Choice of Unitary

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

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Suppose A & B start with state $|\beta_{00}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle_{AB} + |11\rangle_{AB}\right)$

If Alice applies unitary U (2×2 matrix) to her system, and Bob does nothing, what is the total effective unitary on the full 2-qubit system?

A) U_A B) $U_A \otimes I_B + I_A \otimes I_B$ C) $U_A \otimes I_B$ D) $U_A \otimes \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}_B$

$\underbrace{U_A \text{ on system } A, \quad V \text{ on system } B}_{U \otimes V}$

Doing Nothing $= I$ (no rotation)

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A + B start with $|\beta_{00}\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ (1 ebit)

1. Alice applies to her qubit:

$00 \to \mathbb{I}$ or $01 \to X$ or $10 \to Z$ or $11 \to -iY = ZX$ — depending on which bits she wants to send

$\boxed{\text{Q: What happens to her state:}}$

info	U	
00	$\mathbb{I}$	$\to \|\beta_{00}\rangle = \mathbb{I}\otimes\mathbb{I}\|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(\|00\rangle + \|11\rangle)$
01	X	$\to \|\beta_{01}\rangle = X\otimes\mathbb{I}\|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(\|01\rangle + \|10\rangle)$
10	Z	$\to \|\beta_{10}\rangle = Z\otimes\mathbb{I}\|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(\|00\rangle - \|11\rangle)$
11	$-iY$	$\to \|\beta_{11}\rangle = ZX\otimes\mathbb{I}\|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(\|01\rangle - \|10\rangle)$

$$|\beta_{zx}\rangle = Z^z X^x \otimes \mathbb{I}\,|\beta_{00}\rangle$$

2. Alice sends her part of quantum state to Bob (1 qbit)

$\boxed{\text{Q: What measurement should Bob make?}}$

These states form an orthonormal basis. Use this basis for measurement & can distinguish 4 cases perfectly (and learn 2 classical bits)

$$\left\{ \begin{array}{l} |\beta_{00}\rangle \\ |\beta_{01}\rangle \\ |\beta_{10}\rangle \\ |\beta_{11}\rangle \end{array} \right\}$$

Exercise: Check $\langle\beta_{zx}|\beta_{z'x'}\rangle = \delta_{z,z'}\,\delta_{x,x'}$

"Bell Basis"