S. KIMMEL

Announcements

- Last Week
- Tea

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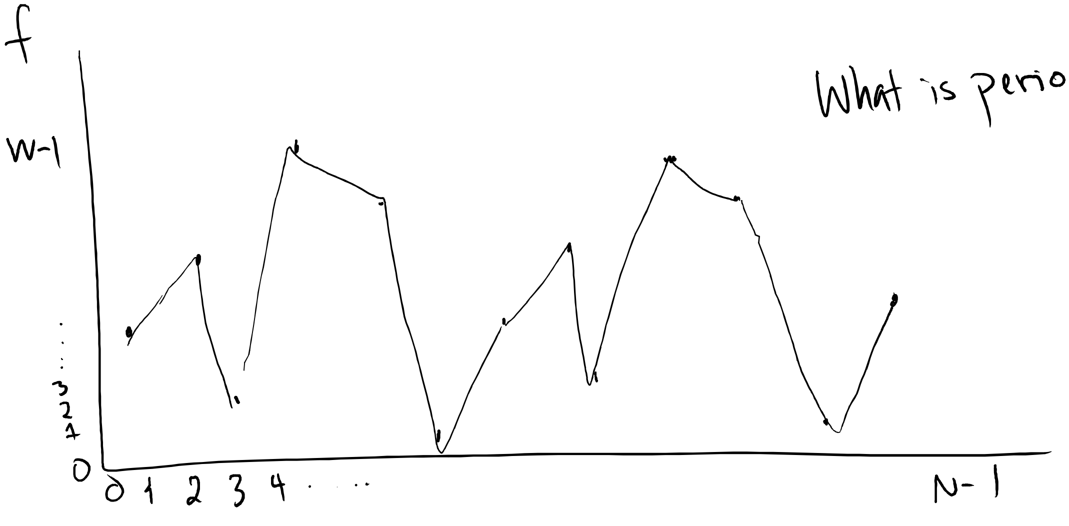
Period Finding Problem

- f has domain $[N]$. Notation: $[N] = \{0, 1, 2, \dots N-1\}$
- Range of f is $[W]$, In other words: $f: [N] \rightarrow [W]$
- f periodic period $r \Rightarrow f(x) = f(x+r)$
- no repeats within a period: $\left(f(i) \neq f(j) \text{ if } |i-j| < r\right)$
- $N > r^2$



What is period?

What is classical query complexity of period finding?

$$\text{A. } O(\log r) \qquad \text{B. } \underset{\Uparrow}{O(r)} \qquad \text{C. } O(r^2) \qquad O(N)$$

can start asking in order until

- Let U_f act on $N \times R$ dimensional quantum system

$$U_f|\underset{\substack{\uparrow\\ N\text{-dim}}}{x}\rangle|\underset{\substack{\uparrow\\ W\text{-dim}}}{y}\rangle = |x\rangle|y+f(x) \bmod W\rangle$$

* Changing standard basis labels:

$$\begin{array}{ccccc} & \text{Old Label} & \text{Vector} & \text{New Label} & \\ & |00\rangle = & \begin{pmatrix}1\\0\\0\\0\end{pmatrix} & = |0\rangle & \\ \text{Binary Rep} \Rightarrow & |01\rangle = & \begin{pmatrix}0\\1\\0\\0\end{pmatrix} & = |1\rangle & \Leftarrow \text{Base 10 Rep} \\ & |10\rangle = & \begin{pmatrix}0\\0\\1\\0\end{pmatrix} & = |2\rangle & \end{array}$$

$$f:\underset{\substack{\uparrow\\ \text{domain}}}{[100]} \to \underset{\substack{\uparrow\\ \text{range}}}{[50]} \qquad \text{Suppose } f(5)=23$$

$$U_f|5\rangle|30\rangle = |5\rangle\,|30+23 \mod 50\rangle$$

$$= |5\rangle\,|3\rangle$$

$$= \underset{\text{length }100\,\rightarrow}{} \begin{pmatrix} 0\\0\\0\\0\\ \underline{1}\\0\\ \vdots \end{pmatrix} \otimes \begin{pmatrix} 0\\0\\ \underline{1}\\0\\0\\ \vdots \end{pmatrix} \leftarrow \text{length } 50$$

Important Unitary: Quantum Fourier Transform for Period Finding

QFT_t is an $t \times t$ unitary (acts on t-dim state)

$$QFT_t : |x\rangle \rightarrow \frac{1}{\sqrt{t}} \sum_{y=0}^{-1} e^{2\pi i \frac{x}{t} y} |y\rangle$$

Q: If apply QFT_t to a standard basis state $|x\rangle$ and then measure in standard basis, what is the probability of getting outcome y:

[✓ A) $\frac{1}{t}$] B) $\frac{1}{\sqrt{t}}$ C) $\frac{xy}{t}$ d $\frac{y}{t}$

Because $\left| \frac{e^{2\pi i \frac{xy}{t}}}{\sqrt{t}} \right|^2 = \left| \frac{1}{\sqrt{t}} \right|^2 \left| e^{2\pi i xy/t} \right|^2 = \frac{1}{t}$

Inverse of QFT

$$QFT_t^{-1} |x\rangle \rightarrow \frac{1}{\sqrt{t}} \sum_{y=0}^{t-1} e^{-2\pi i \frac{x}{t} y} |y\rangle$$

Basic Algorithm:

1. Prepare $\underset{\text{N-dim}}{|0\rangle_A}\underset{\text{W-dim}}{|0\rangle_B}$

2. Apply QFT_N to A

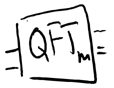
3. Apply U_f to A, B

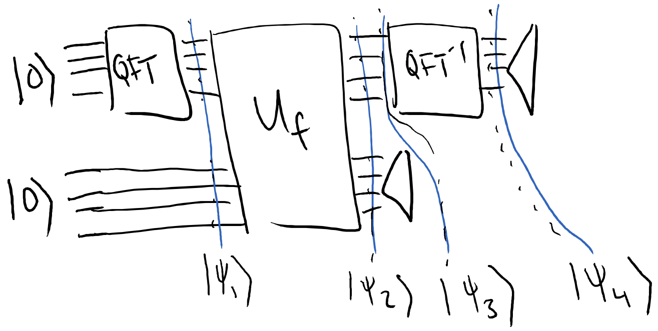
4. Measure B in standard basis

5. Apply QFT_N^{-1} to A

6. Measure A in standard basis

Q: Write as circuit





Total Algorithm

1. Run basic algorithm twice. Get outcomes y, y'.

Do Classical postprocessing on y, y'. Outcome is pretty likely to be $r \rightarrow$ can check if outcome is correct