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Today: Quantum States & Measurements

Last time: Questions about quant. Crypto?

Announcements
- PS due Wed

Representing States with Vectors:

$\updownarrow = \begin{pmatrix}1\\0\end{pmatrix} = |0\rangle$

$\leftrightarrow = \begin{pmatrix}0\\1\end{pmatrix} = |1\rangle$

$\nearrow\!\!\!\swarrow = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} = |+\rangle$

$\nwarrow\!\!\!\searrow = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} = |-\rangle$

Circularly polarized. Used for 3D movies

$\circlearrowright = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix} = |\rightarrow\rangle$

$\circlearrowleft = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix} = |\leftarrow\rangle$

Q. How many possible qubit states are there? $(2, 6, \infty)$

A. ∞

Qubit States

$\begin{pmatrix}a_0\\a_1\end{pmatrix}$, $a_0, a_1 \in \mathbb{C}$ ← complex numbers, "amplitudes"

$\boxed{|a_0|^2 + |a_1|^2 = 1}$ ← "normalization"

Bit has two states: 0, 1. Qubit has two "standard basis" states:

$\begin{pmatrix}1\\0\end{pmatrix}$ = "zero state" $= |0\rangle$ "ket notation"

$\begin{pmatrix}0\\1\end{pmatrix}$ = "one state" $= |1\rangle$

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Common to write

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

"Superposition" = linear combination

"$|+\rangle$ is in a superposition of $|0\rangle$ and $|1\rangle$"

Note: $\underbrace{|+\rangle, |-\rangle}_{\text{basis}}$

So $$|0\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right)$$

"$|0\rangle$ is in a superposition of $|+\rangle$ and $|-\rangle$"

Whether a state is a superposition or not depends on your basis. However, when we say superposition, we usually mean relative to standard basis

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Bras & Kets

In Linear Algebra class: $\vec{X} = \begin{pmatrix} x_0 \\ x_1 \\ \vdots \\ x_k \end{pmatrix}$ $\quad \vec{X}^{\overset{\text{conjugate transpose}}{\dagger}} = (x_0^*, x_1^* \cdots \; x_k^*)$ ← complex conjugate

In Quantum computing: $\underbrace{|\psi\rangle}_{\text{"ket psi"}} = \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$ $\quad \underbrace{\langle\psi|}_{\text{"bra psi"}} = (a_0^* \; a_1^*)$

Inner Product

$|\psi\rangle = a_0|0\rangle + a_1|1\rangle$

$|\phi\rangle = b_0|0\rangle + b_1|1\rangle$

$$\underbrace{\langle\phi|\psi\rangle}_{\text{"inner product"}} = (b_0^*, b_1^*)\begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = b_0^* a_0 + b_1^* a_1$$

Using bra/kets: $\langle\phi|\psi\rangle = \left(b_0^*\langle 0| + b_1^*\langle 1|\right)\left(a_0|0\rangle + a_1|1\rangle\right) = a_0 b_0^* \overbrace{\langle 0|0\rangle}^{1} + b_0^* a_1 \overbrace{\langle 0|1\rangle}^{0} + b_1^* a_0 \overbrace{\langle 1|0\rangle}^{0} + b_1^* a_1 \overbrace{\langle 1|1\rangle}^{1}$

Whenever you see inner product of standard basis states, is 0 if different, 1 if same!

Q: For a qubit state $|\psi\rangle = a_0|0\rangle + a_1|1\rangle$, what is $\langle\psi|\psi\rangle$?

$$\langle\psi|\psi\rangle = (a_0^* \; a_1^*)\begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = a_0^* a_0 + a_1^* a_1 = |a_0|^2 + |a_1|^2 = 1$$

$\boxed{\text{Inner product of quantum state with itself is } 1}$

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2. Qubit Measurement (von Neumann Measurement)

Measurement is represented by an orthonormal basis: $M = \{|\phi_0\rangle, |\phi_1\rangle\}$

- 2 states $|\phi_0\rangle, |\phi_1\rangle$ s.t. $\underbrace{\langle\phi_0|\phi_1\rangle = 0}_{\text{orthogonal}}$, and $\underbrace{\langle\phi_0|\phi_0\rangle = \langle\phi_1|\phi_1\rangle = 1}_{\text{normalized}}$

If initially, system is $|\psi\rangle$,

- with probability $|\langle\phi_0|\psi\rangle|^2$, get outcome $|\phi_0\rangle$, state becomes $|\phi_0\rangle$
- with probability $|\langle\phi_1|\psi\rangle|^2$, get outcome $|\phi_1\rangle$, state becomes $|\phi_1\rangle$

We say "state $|\psi\rangle$ collapses to $|\phi_0\rangle$ or $|\phi_1\rangle$"