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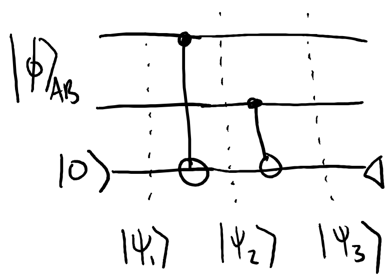
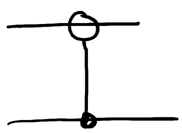
Practice With Q. Circuits

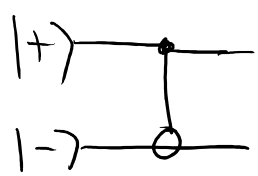
1. What does this circuit do?
   - When $|\phi_{AB}\rangle$ is standard basis state?
   - When $|\phi_{AB}\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$



Hint: calculate each intermediate state $|\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle$

2. Write  in matrix form (CNOT with order reverse)

3. What is the output of this circuit in the $\{|+\rangle, |-\rangle\}$ basis?



Solutions on next page.

Goals

Analyze Q. Circuits

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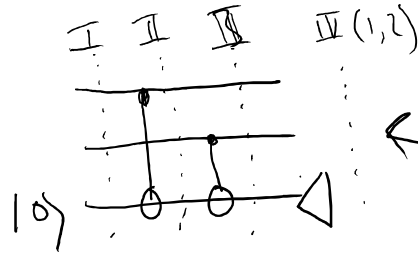
1. What does this circuit do?



arbitrary input

$a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle$

interpret output based on whether measurement is 0,1

$$a_{00}|000\rangle + a_{01}|010\rangle + a_{10}|100\rangle + a_{11}|110\rangle$$

$$a_{00}|000\rangle + a_{01}|010\rangle + a_{10}|101\rangle + a_{11}|111\rangle$$

$$a_{00}|000\rangle + a_{01}|011\rangle + a_{10}|101\rangle + a_{11}|110\rangle$$

$$\left(a_{00}|00\rangle + a_{11}|11\rangle\right)|0\rangle + \left(a_{01}|01\rangle + a_{10}|10\rangle\right)|1\rangle$$

$\downarrow$ measure $|0\rangle$ $\qquad\qquad$ $\downarrow$ measure $|1\rangle$

$$\frac{a_{00}|00\rangle + a_{11}|11\rangle}{\sqrt{|a_{00}|^2 + |a_{11}|^2}} \qquad\qquad \frac{a_{01}|01\rangle + a_{10}|10\rangle}{\sqrt{|a_{01}|^2 + |a_{10}|^2}}$$

$\Uparrow$ $\qquad\qquad$ $\Uparrow$

Collapse to even parity part $\qquad\qquad$ collapse to odd parity part

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2. $U_{AB} = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X$

$$U_{BA} = I \otimes |0\rangle\langle 0| + X \otimes |1\rangle\langle 1|$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

3. $U|+\rangle|-\rangle = \left(|0\rangle\langle 0| \otimes I\right)|+\rangle|-\rangle + \left(|1\rangle\langle 1| \otimes X\right)|+\rangle|-\rangle$

$$= |0\rangle\langle 0|+\rangle \otimes I|-\rangle + |1\rangle\langle 1|+\rangle \otimes X|-\rangle$$

$$= \frac{1}{\sqrt{2}}|0\rangle|-\rangle - \frac{1}{\sqrt{2}}|1\rangle|-\rangle$$

$$= |-\rangle|-\rangle$$

<u>First Qubit Changes!</u>

U seems like it only changes second system, but in $\{|+\rangle, |-\rangle\}$ basis, second qubit stays the same, and first flips!

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Basic Quantum Algorithms

Goal:

- Strategies for analyzing quantum algorithms
- Understanding of why quantum algs. can do better than quantum.

Deutsch Algorithm

Consider a one bit function f:

x	f(x)
0	?
1	?

Options for f:

x	f(x)
0	0
1	0

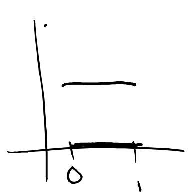
x	f(x)
X	1
1	1

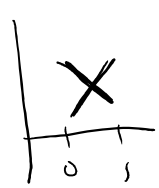
"even"

x	f(x)
0	0
1	1

x	f(x)
0	1
1	0

"balanced"





Problem: Decide if f is even or balanced.

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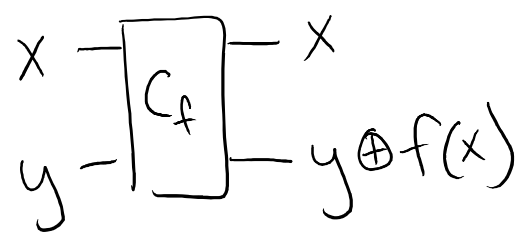
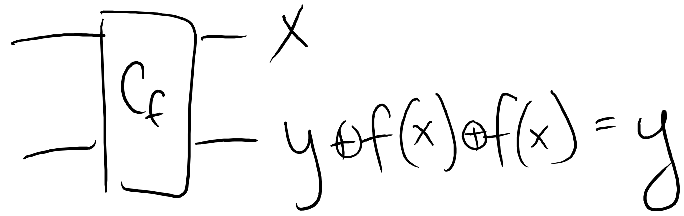
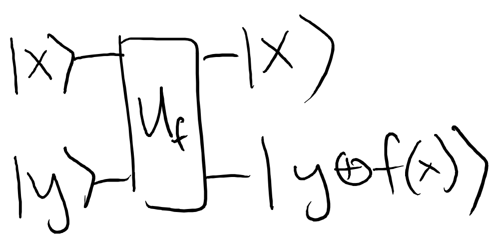
How is f given? Reversible f (classical)





Reversible f (quantum)



← $|x\rangle, |y\rangle$ are standard basis states. So if $|x\rangle, |y\rangle$ are standard basis states, acts just like classical.

Problem (precise): How many uses of C_f / U_f are required as part of a classical/quantum circuit to determine if f is balanced or even?

classical answer

quantum answer

<u>Classical Query Complexity</u>

<u>Quantum Query Complexity</u>