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Projectors in Shor's 9-qubit code

$|0_L\rangle = \big(|000\rangle+|111\rangle\big)\big(|000\rangle+|111\rangle\big)\big(|000\rangle+|111\rangle\big)$

$|1_L\rangle = \big(|000\rangle-|111\rangle\big)\big(|000\rangle-|111\rangle\big)\big(|000\rangle-|111\rangle\big)$

$P_0 = |0_L\rangle\langle 0_L| + |1_L\rangle\langle 1_L|$

$P_{X_1} = X_1|0_L\rangle\langle 0_L|X_1 + X_1|1_L\rangle\langle 1_L|X_1$

$X_1|0_L\rangle = \big(|100\rangle+|011\rangle\big)\big(|000\rangle+|111\rangle\big)\big(|000\rangle+|111\rangle\big)$

$X_1|1_L\rangle \; \big(|100\rangle-|011\rangle\big)\big(|000\rangle-|111\rangle\big)\big(|000\rangle-|111\rangle\big)$

Key: $P_0 P_{X_1} = 0$ (b/c $\langle 0_L|(X|0_L\rangle = 0$
$\langle 0_L|X|1_L\rangle = 0$
$\langle 1_L|X|0_L\rangle = 0$)

So X_1 takes element of code to orthogonal subspace (different "bubble")

Q: Which of the following errors can be accurately corrected by Shor's 9-qubit code? Code corrects by assuming fewest number of errors possible occured

A) Z_1Z_2

B) Z_1Z_4

C) $Z_1Z_2Z_3$

D) X_1Z_2

E) X_1X_2

F) X_1X_4

G) Y_1Z_4

All these errors take code to ortho. subspace. Issue is whether fewer # of errors could do same thing

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✓ A) $Z_1 Z_2$

✗ B) $Z_1 Z_4$

✓ C) $Z_1 Z_2 Z_3$

✓ D) $X_1 Z_2$

✗ E) $X_1 X_2$

✓ F) $X_1 X_4$

✗ G) $Y_1 Z_4$

Issues

Threshold

- Shor Code: corrects 1 error
- Need to complete correction circuit before 2^{nd} error occurs
- Current error rates too high. Need to be below some critical rate: threshold $\boxed{\sim 10^{-4} \text{ prob of error}}$

getting close

Gates

- Need to apply gates to logical qubits
- If error occurs, gate implementation circuit can make error spread.
- H, CNOT: OK, $\boxed{T = \text{NOT OK}}$ ← we have ways to deal with this, but not great

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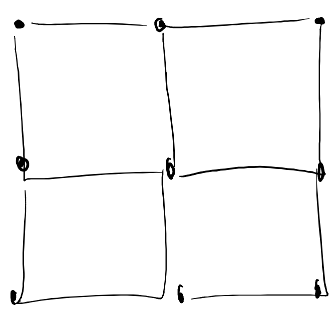
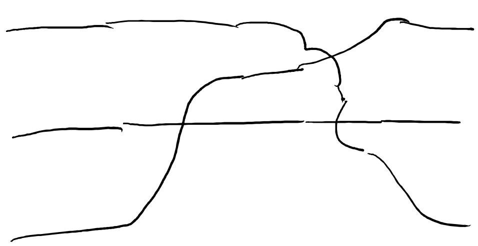
Current Research in Codes

- Flag qubits:
  - 19 physical qubits (including ancillas)
  - 7 logical qubits
  - Protects against 1 qubit errors
- Color Codes



- qubits usually on a grid
- easier to apply CNOT on edges
- color codes are easier to implement respecting this locality

- Topological codes (* I don't buy)



- gates implemented by braiding
- errors only result from braids → random errors unlikely to braid if strands kept far apart
- Need physical systems that might not exist

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