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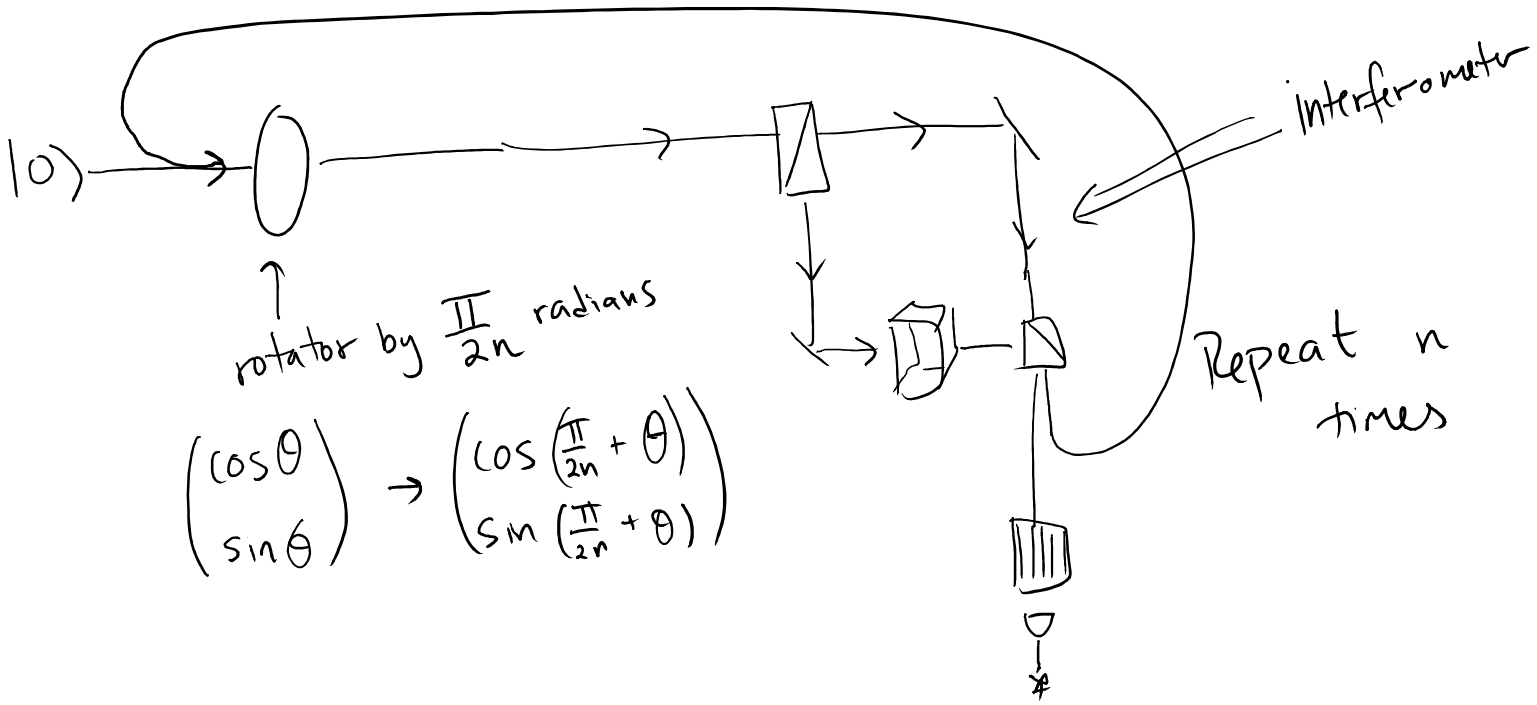

1) If no bomb, what is prob. of photon detection?

- Same state enters interferometer as leaves
- Adds $\frac{\pi}{2n}$ angles n times $\Rightarrow \frac{\pi}{2}$
- Initially $\theta = 0$ $\quad \begin{pmatrix}\cos 0\\ \sin 0\end{pmatrix} = \begin{pmatrix}1\\ 0\end{pmatrix} \Rightarrow \begin{pmatrix}\cos\frac{\pi}{2}\\ \sin\frac{\pi}{2}\end{pmatrix} = \begin{pmatrix}0\\ 1\end{pmatrix}$
- Will be blocked by filter $\Rightarrow$ No detection

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2) If bomb, what is prob of No explosion?

- Each time no explosion, state collapses to $|0\rangle$.
- Rotates $\Rightarrow \begin{pmatrix} \cos\frac{\pi}{2n} \\ \sin\frac{\pi}{2n} \end{pmatrix}$
- P_r(No explosion in j^{th} round | no explosion yet) = $\cos^2\frac{\pi}{2n}$
- P_r(no explosion) = $\left(\cos\left(\frac{\pi}{2n}\right)\right)^{2n}$

3) If bomb but no explosion, what is the probability of photon detection?

- If bomb doesn't explode, state keeps collapsing to $|0\rangle$, so $|0\rangle$ exits, and you get a detection with probability 1.

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Taylor Series Expansion

If $x \ll 1$: $\cos(x) \approx 1 - \frac{x^2}{2}$

$$(1+x)^k \approx 1 + kx$$

If $n \gg 1$: $\left(\cos\left(\frac{\pi}{2n}\right)\right)^{2n} \approx \left(1 - \frac{\pi^2}{8n^2}\right)^{2n} \approx 1 - \frac{\pi^2}{4n}$

$\Rightarrow$ Probability of Bomb detonating is small, so can figure out if live or dud!

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Quantum Operations (gates)

Reversible transformations

Mathematical representation: (qubit operation)

Unitary matrix $U \in \mathbb{C}^{2\times 2}$

(2×2 complex matrix)

$$U U^\dagger = U^\dagger U = I$$

$U^\dagger$: conj. transpose

I: identity $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

In action

$$|\psi\rangle \xrightarrow{U \text{ acts}} U|\psi\rangle$$

(matrix multiplication)

$|\psi\rangle$: vector

$U|\psi\rangle$: matrix · vector = vector