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# Alice's measurement: Bell Basis: $\{|\beta_{00}\rangle, |\beta_{01}\rangle, |\beta_{10}\rangle, |\beta_{11}\rangle\}$

$$\left.\begin{aligned}
|\beta_{00}\rangle &= \mathbb{I}\otimes\mathbb{I}\,|\beta_{00}\rangle = \tfrac{1}{\sqrt{2}}(|00\rangle+|11\rangle)\\
|\beta_{01}\rangle &= X\otimes\mathbb{I}\,|\beta_{00}\rangle = \tfrac{1}{\sqrt{2}}(|01\rangle+|10\rangle)\\
|\beta_{10}\rangle &= Z\otimes\mathbb{I}\,|\beta_{00}\rangle = \tfrac{1}{\sqrt{2}}(|00\rangle-|11\rangle)\\
|\beta_{11}\rangle &= ZX\otimes\mathbb{I}\,|\beta_{00}\rangle = \tfrac{1}{\sqrt{2}}(|01\rangle-|10\rangle)
\end{aligned}\right\}$$

Each of these outcomes is a maximally entangled state

## Strategy

state to send $= a|0\rangle+b|1\rangle$

ebit shared

1. A & B start with $|\psi\rangle_{A_1}|\beta_{00}\rangle_{A_2B}$ (1 ebit)
2. Alice measures $A_1$ and $A_2$ (this destroys entanglement) using Bell Basis
3. Alice sends classical outcome (00, 01, 10, or 11) to Bob (2 cbits)
4. Bob applies a unitary to his system B based on Alice's cbits

What state does Bob end up with in each case?

Which unitary should Bob apply for each outcome?

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1. Alice measures qubits $A_1$ & $A_2$ in Bell basis (partial measurement

$$|\psi\rangle_{A_1}|\beta_{00}\rangle_{A_2B} = (a|0\rangle+b|1\rangle)\otimes\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$$
$$= \frac{1}{\sqrt{2}}\left(a|000\rangle + a|011\rangle + b|100\rangle + b|111\rangle\right)_{A_1A_2B}$$

$$|\psi\rangle|\beta_{00}\rangle = I_{A_1A_2}\otimes I_B|\psi\rangle|\beta_{00}\rangle =$$

$$= \underbrace{\left(|\beta_{00}\rangle\langle\beta_{00}| + |\beta_{01}\rangle\langle\beta_{01}| + |\beta_{10}\rangle\langle\beta_{10}| + |\beta_{11}\rangle\langle\beta_{11}|\right)_{A_1A_2}}_{I_{A_1A_2}}\otimes I_B|\psi\rangle_{A_1}|\beta_{00}\rangle_{A_2B}$$

Calculation for $|\beta_{00}\rangle\langle\beta_{00}|$ term

$$|\beta_{00}\rangle\langle\beta_{00}|_{A_1A_2}\otimes I_B\,\frac{1}{\sqrt{2}}\left(a|000\rangle + a|011\rangle + b|100\rangle + b|111\rangle\right)_{A_1A_2B}$$
$$= \frac{1}{\sqrt{2}}\Big[a\,|\beta_{00}\rangle\langle\beta_{00}|00\rangle_{A_1A_2}I_B|0\rangle_B + a|\beta_{00}\rangle\langle\beta_{00}|01\rangle_{A_1A_2}I|1\rangle$$
$$+ b\,|\beta_{00}\rangle\langle\beta_{00}|10\rangle_{A_1A_2}|0\rangle_B + b|\beta_{00}\rangle\langle\beta_{00}|11\rangle_{A_1A_2}I|1\rangle$$

Commute tems to bring together those with same subscript

$$= \frac{1}{\sqrt{2}}\Big[a\cdot\frac{1}{\sqrt{2}}|\beta_{00}\rangle_{A_1A_2}|0\rangle_B + 0 + 0 +$$
$$b\,\frac{1}{\sqrt{2}}|\beta_{00}\rangle_{A_1A_2}|1\rangle_B\Big]$$

Use

$$\boxed{\begin{aligned}\langle\beta_{00}|00\rangle &= \tfrac{1}{\sqrt{2}}\\ \langle\beta_{00}|10\rangle &= 0\\ \langle\beta_{00}|01\rangle &= 0\\ \langle\beta_{00}|11\rangle &= \tfrac{1}{\sqrt{2}}\end{aligned}}$$

$$= \underset{\text{amplitude}}{\frac{1}{2}}\;\underset{\text{measurement outcome}}{|\beta_{00}\rangle_{A_1A_2}}\;\underset{\text{normalized state}}{(a|0\rangle+b|1\rangle)_B}$$

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If do same process for other terms:

$$|\psi\rangle_{A_1}|\beta_{00}\rangle_{A_2B} = (a|0\rangle+b|1\rangle)_{A_1}|\beta_{00}\rangle_{A_2B}$$

$$= \frac{1}{2}|\beta_{00}\rangle_{A_1A_2}(a|0\rangle+b|1\rangle)_B$$

$$+ \frac{1}{2}|\beta_{01}\rangle_{A_1A_2}(b|0\rangle+a|1\rangle)_B$$

$$+ \frac{1}{2}|\beta_{10}\rangle_{A_1A_2}(a|0\rangle-b|1\rangle)_B$$

$$+ \frac{1}{2}|\beta_{11}\rangle_{A_1A_2}(b|0\rangle-a|1\rangle)_B$$

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| Alice's Outcome | Bob's State | |
|---|---|---|
| $\|\beta_{00}\rangle$ | $\|\psi\rangle$ | $= a\|0\rangle + b\|1\rangle$ |
| $\|\beta_{01}\rangle$ | $X\|\psi\rangle$ | $= b\|0\rangle + a\|1\rangle$ |
| $\|\beta_{10}\rangle$ | $Z\|\psi\rangle$ | $= a\|0\rangle - b\|1\rangle$ |
| $\|\beta_{11}\rangle$ | $ZX\|\psi\rangle$ | $= b\|0\rangle - a\|1\rangle$ |

2. Alice uses 2 cbits to tell Bob her outcome
3. Bob uses cbit info to determine recovery operation:

$$
\begin{aligned}
00 &\rightarrow \text{do nothing} \\
01 &\rightarrow \text{apply } X^{-1} = X \\
10 &\rightarrow \text{apply } Z^{-1} = Z \\
11 &\rightarrow \text{apply } (ZX)^{-1} = XZ
\end{aligned}
$$

## Big Picture

Alice's $A_1$ qubit had state $a|0\rangle + b|1\rangle$. This is now the state of Bob's qubit. She never communicated $a, b$. It just shows up in Bob's possession.

So... Beam Me Up, Scottie!

But: need an astronomical amount of entanglement for macroscopic object. Also info might be there, but might not be in correct form.