CS333 - Gates Worksheet

1. We have three ways of representing unitaries: in matrix form, in terms of how it transforms standard basis states, and in ket-bra form. I will give you a unitary in one of the forms; please write its representation using the other two forms, and also verify that it is indeed unitary.

   (a)

$$CP = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & i \end{pmatrix} \tag{1}$$

   (b)

$$\begin{aligned} U|0\rangle =& \frac{1}{\sqrt{2}} \left(|0\rangle + i|1\rangle\right), \\ U|1\rangle =& \frac{1}{\sqrt{2}} \left(-i|0\rangle - |1\rangle\right) \end{aligned} \tag{2}$$

   (c) $V = |00\rangle\langle 00| + |11\rangle\langle 11| + \frac{1}{\sqrt{2}} \left(|01\rangle\langle 01| + |01\rangle\langle 10| + |10\rangle\langle 01| - |10\rangle\langle 10|\right)$

2. If U is a unitary and $|\psi\rangle$ is a quantum state, is $U|\psi\rangle$ always a state? (Think about what our criteria are for quantum states.)

3. I said the unitary operation U is reversible. This means there is another unitary matrix V that undoes the action of U. What is this other matrix and how do you know it is a unitary (and hence a valid quantum operation)?