CS333 - Final Review

1. Consider the following two-qubit state:

$$\frac{1}{\sqrt{3}}\left(|0\rangle|+\rangle + |-\rangle|1\rangle\right) \tag{1}$$

   (a) **[3 points]** If you measure the first qubit of $|\psi\rangle$ using the basis $\{|0\rangle, |1\rangle\}$, what is the probability of getting outcome $|0\rangle$?

   (b) **[3 points]** If you get outcome $|0\rangle$ on the first qubit, what state will the second qubit collapse to?

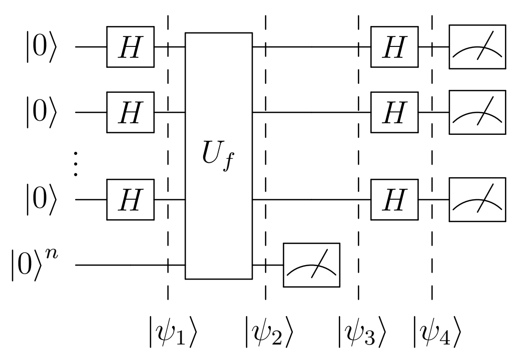
   (c) **[3 points]** If you measure the first qubit of $|\psi\rangle$ using the basis $\{|+\rangle, |-\rangle\}$, what is the probability of getting outcome $|0\rangle$?

   (d) **[3 points]** If you get outcome $|+\rangle$ on the first qubit, what state will the second qubit collapse to?

2. Suppose you have a function $f : \{0,1\}^n \to \{0,1\}^n$, where $f(x) = f(y)$ if and only if $x = y \oplus s$ for some $s \in \{0,1\}^n$. (Here $\oplus$ means addition mod 2 for each bit of the string.) Otherwise, there is no structure in terms of which input gets assigned to which output. Recall for standard basis states: $H^{\otimes n}|x\rangle = \sum_{y=0}^{2^n-1}(-1)^{x\cdot y}|y\rangle$, where $x \cdot y = \sum_{j=1}^{n} x_j y_j$ where x_j is the jth bit of x and y_j is the jth bit of y.

   (a) What is the classical query complexity of determining s?

   (b) Suppose you have a unitary that acts as $U_f|x\rangle|0\rangle = |x\rangle|f(x)\rangle$. (For this algorithm, it doesn't matter how U_f acts on other standard basis states.) If we run the following algorithm:



(2)

What are the states at each point?

(c) If you can find $O(n)$ (randomly chosen) z such that $z \cdot s = 0 \mod 2$, then can figure out s. Use this fact to determine the quantum query complexity of learning s.

3. I said that there is a similar code to the 3-qubit bit-flip code we looked at in class that corrects Z errors. The idea is to convert from the $|0\rangle/|1\rangle$ view to a $|+\rangle/|-\rangle$ view, since $|+\rangle/|-\rangle$ are sensitive to Z errors, but the approach otherwise should be similar to the 3-qubit bit-flip code.

   (a) Draw a circuit that encodes a qubit $a|0\rangle + b|1\rangle$ into a 3-qubit state that corrects against Z errors. (Your circuit should use standard gates like H, $CNOT$, etc.)

   (b) Show how to detect Z errors using two ancillary qubits, control operations, and a measurement of the ancillary qubits.

   (c) What is the projective measurement that your circuit in part (b) accomplishes?

Hints on 1:

- $(a \oplus b) \cdot c \equiv a \cdot c + b \cdot c \mod 2$.
- There should be two standard basis states in superposition after the measurement of the second register.

Hints on 2:

- Convert the initial state to $|+\rangle/|-\rangle$ basis. Then think about how you can convert the $|+\rangle$ into $|+++\rangle$ and the $|-\rangle$ into $|---\rangle$ similarly to what we do in the bit-flip case.