S. KIMMEL

Search Problem

Given $f: \{0,1,2,3,\ldots,N-1\} \to \{0,1\}$, promised exactly 1 s such that $f(s) = 1$

("marked element")

- Classical Case:
  Given classical f, what is deterministic / probabilistic query complexity?

A) N, $O(N/\log(N))$

B) $N-1$, $O(N)$

C) $N-1$, $O(N/\log(N))$

D) N, $O(N)$

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On average, have to query half $(2^n/2)$ before finding marked item.

Adversary will choose to put "marked" input where won't query until end

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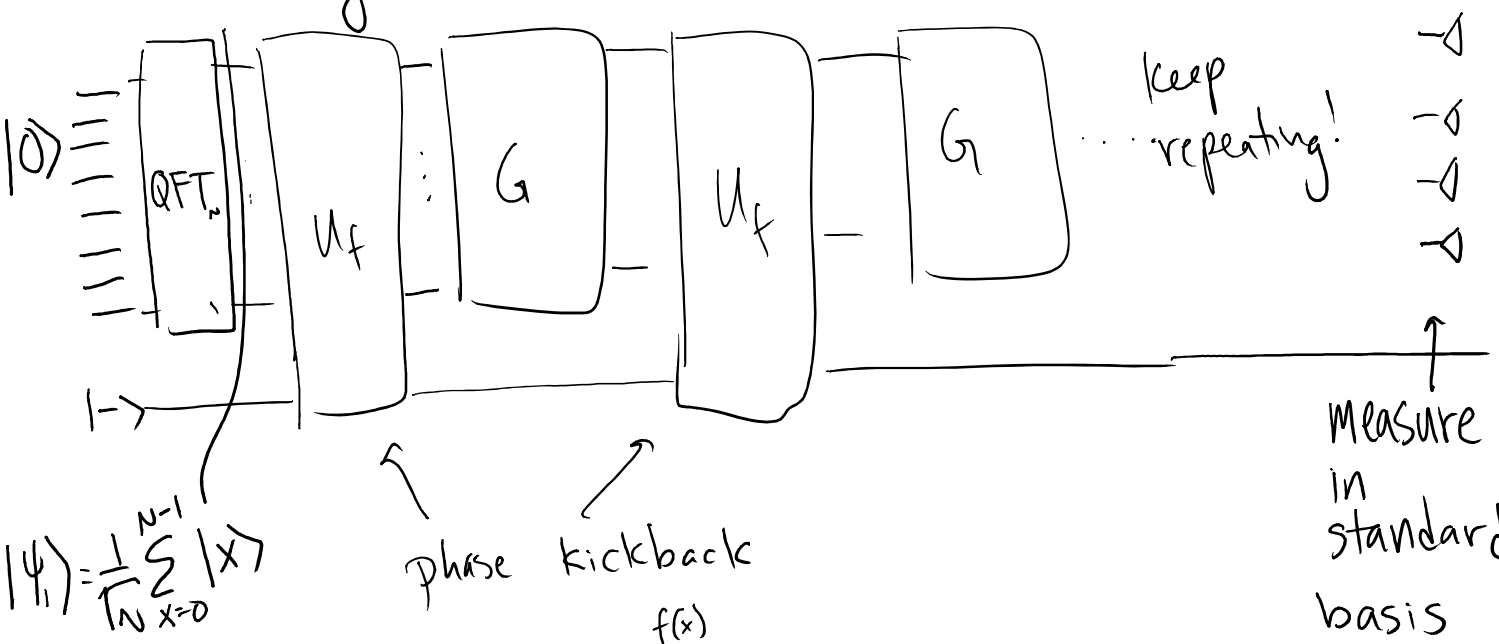
Quantum Alg: (Grover's Algorithm)



$|\psi_1\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle$

phase kickback

$|x\rangle|-\rangle \rightarrow (-1)^{f(x)}|x\rangle|-\rangle$

U_f: $\mathbb{I} - 2|s\rangle\langle s|$ (applies -1 to $|s\rangle$, $+1$ to all other standard basis states)

$$U_f = \left(\sum_i |i\rangle\langle i|\right) - 2|s\rangle\langle s| = \left(\sum_{i \neq s} |i\rangle\langle i|\right) - |s\rangle\langle s|$$

G_f: $-\mathbb{I} + 2|\alpha\rangle\langle\alpha|$ where $|\alpha\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle$

applies $+1$ to $|\alpha\rangle$ and -1 to all orthogonal states

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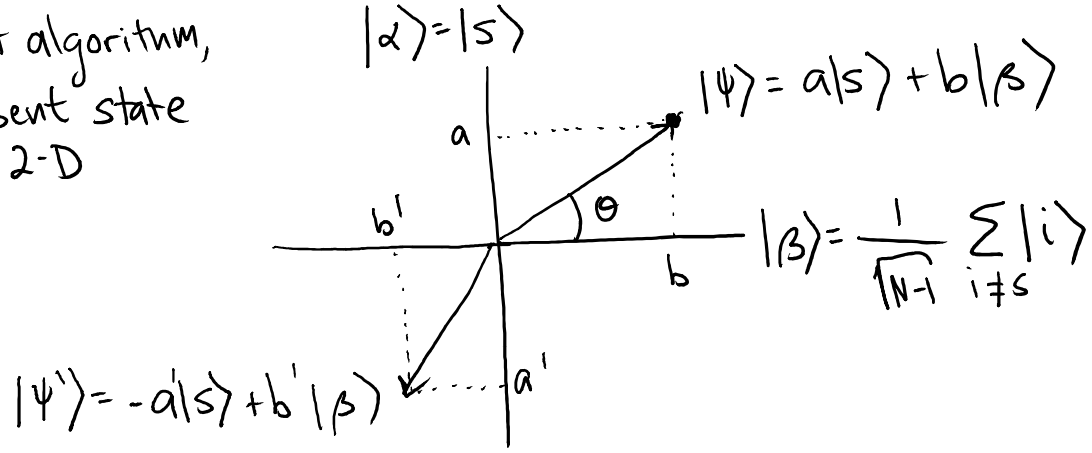
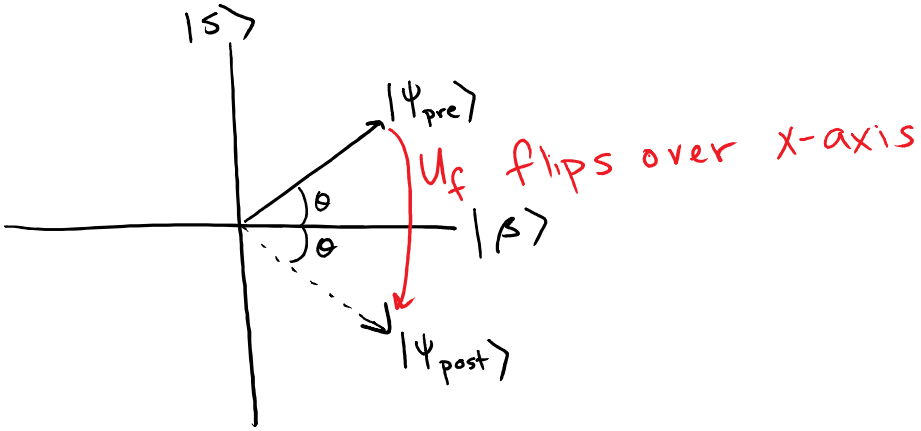
Throughout algorithm, can represent state using a 2-D axis



Effect of U_f: Adds -1 phase to $|s\rangle$



Effect of G: Adds -1 phase to anything orthogonal to $|\alpha\rangle$