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Learning Goals

- Analyze qubit behavior using vectors.
- Describe CHSH game.

- Qubit State $\rightarrow \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} \quad |a_0|^2 + |a_1|^2 = 1$

"ket" $\Rightarrow |\psi\rangle = a_0|0\rangle + a_1|1\rangle \rightarrow a_0\begin{pmatrix} 1 \\ 0 \end{pmatrix} + a_1\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$

$|\phi\rangle = b|+\rangle + c|1\rangle \rightarrow \frac{b}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + c\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{b}{\sqrt{2}} \\ \frac{b}{\sqrt{2}} + c \end{pmatrix}$

- Inner product of $|\psi\rangle$ and $|\phi\rangle$:

$$\langle\phi|\psi\rangle = \underbrace{\left(\frac{b^*}{\sqrt{2}}, \frac{b^*}{\sqrt{2}} + c^*\right)}_{} \cdot \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$$

"bra" $\Rightarrow \langle\phi| =$ conjugate transpose of $|\phi\rangle$

(use matrix multiplication) $= \frac{a_0 b^*}{\sqrt{2}} + \frac{a_1 b^*}{\sqrt{2}} + a_1 c^*$

Q: What is the inner product of a state $|\psi\rangle$ with itself?

A): 0, B) $\frac{1}{2}$ C) 1 D) Depends on state

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$|\psi\rangle = \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$ $\langle\psi|\psi\rangle = \begin{pmatrix} a_0^* & a_1^* \end{pmatrix}\begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = a_0^* a_0 + a_1^* a_1$

$= |a_0|^2 + |a_1|^2 = 1$

Alternative Approach to Inner Product using bras/kets

$$\langle\phi|\psi\rangle = \left(b^*\langle +| + c^*\langle 1|\right)\left(a_0|0\rangle + a_1|1\rangle\right)$$

(distribute) $= b^* a_0 \cancel{\langle +|0\rangle}^{1/\sqrt{2}} + c^* a_0 \cancel{\langle 1|0\rangle}^{0} + b^* a_1 \cancel{\langle +|1\rangle}^{1/\sqrt{2}}$

$+ c^* a_1 \cancel{\langle 1|1\rangle}^{1}$

$= \frac{b^* a_0}{\sqrt{2}} + \frac{b^* a_1}{\sqrt{2}} + c^* a_1$

$\boxed{|0\rangle, |1\rangle \text{ orthonormal! } \langle 0|0\rangle = 1 \quad \langle 0|1\rangle = 0}$

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Qubit Measurement

Represented by: $\underbrace{\text{orthonormal basis}}$: $M = \{|\phi_0\rangle, |\phi_1\rangle\}$

$\underbrace{\langle\phi_0|\phi_1\rangle = 0}_{\text{orthogonal}}$, and $\underbrace{\langle\phi_0|\phi_0\rangle = \langle\phi_1|\phi_1\rangle = 1}_{\text{normalized}}$

If measure state $|\psi\rangle$

- with probability $|\langle\phi_0|\psi\rangle|^2$, get outcome $|\phi_0\rangle$, state becomes $|\phi_0\rangle$
- with probability $|\langle\phi_1|\psi\rangle|^2$, get outcome $|\phi_1\rangle$, state becomes $|\phi_1\rangle$

We say "state $|\psi\rangle$ collapses to $|\phi_0\rangle$ or $|\phi_1\rangle$"

Examples of orthonormal bases:

$\{|0\rangle, |1\rangle\}$, $\{|+\rangle, |-\rangle\}$, $\{|\circlearrowright\rangle, |\circlearrowleft\rangle\}$

$\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}$ $\quad \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$ $\quad \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}$