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Representing States with Vectors:

$$\updownarrow \; = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$$

$$\leftrightarrow \; = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$

$$\nearrow\!\!\!\!\swarrow \; = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = |+\rangle$$

$$\nwarrow\!\!\!\!\searrow \; = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = |-\rangle$$

Circularly polarized. Used for 3D movies

$$\circlearrowright \; = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix} = |\rightarrow\rangle$$

$$\circlearrowleft \; = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix} = |\leftarrow\rangle$$

Q: How many qubit states are there?

A) 2 B) 6 C) Countably infinite D) uncountably infinite

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all possible angles of polarization

Qubit States

$\begin{pmatrix}a_0\\a_1\end{pmatrix}$, $a_0, a_1 \in \mathbb{C}$ complex #'s, "amplitudes"

$\boxed{|a_0|^2 + |a_1|^2 = 1}$ ← "normalization"

Bit : 0, 1. Qubit: $|0\rangle, |1\rangle$ "standard basis"

$\begin{pmatrix}1\\0\end{pmatrix}$ = "zero state" = $|0\rangle$ "ket notation"

$\begin{pmatrix}0\\1\end{pmatrix}$ = "one state" = $|1\rangle$

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Common to write

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

"Superposition" = linear combination

"$|+\rangle$ is in a superposition of $|0\rangle$ and $|1\rangle$"

Note: $\underbrace{|+\rangle, |-\rangle}_{\text{orthonormal basis}}$

So $$|0\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right)$$

"$|0\rangle$ is in a superposition of $|+\rangle$ and $|-\rangle$"

Whether a state is a superposition or not depends on your basis. However, when we say superposition, we usually mean relative to standard basis

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Bras & Kets

conjugate transpose

Linear algebra: $\vec{x} = \begin{pmatrix} x_0 \\ x_1 \\ \vdots \\ x_k \end{pmatrix}$ $\quad \vec{x}^\dagger = (x_0^*, x_1^* \cdots, x_k^*)$ complex conjugate

Quantum Computing: $|\psi\rangle = \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$ $\quad \langle\psi| = (a_0^* \;\; a_1^*)$

"ket psi" "bra psi"

Useful fact $\quad |\psi\rangle = a|\phi_0\rangle + b|\phi_1\rangle \quad (a, b \in \mathbb{C},\ |\phi_0\rangle, |\phi_1\rangle \in \mathbb{C}^2)$

$$\langle\psi| = a^*\langle\phi_0| + b^*\langle\phi_1|$$