

Representing States with Vectors:

$$\downarrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$$

$$\leftarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$

$$\nearrow = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = |+\rangle$$

$$\searrow = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = |-\rangle$$

Circularly
polarized.
Used for
3D movies

$$\left\{ \begin{array}{l} \odot = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} = | \rightarrow \rangle \\ \otimes = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} = | \leftarrow \rangle \end{array} \right.$$

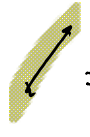
Q: How many qubit states are there?

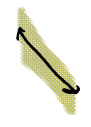
- A) 2 B) 6 C) countably infinite D) uncountably infinite

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Circularly polarized. Used for 3D movies

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all possible angles of polarization

Q: How many qubit states are there?

- A) 2 B) 6 C) countably infinite **D) uncountably infinite**

Qubit States

$\begin{pmatrix} a_0 \\ a_1 \end{pmatrix}, a_0, a_1 \in \mathbb{C}$ complex #'s, "amplitudes" $|a_0|^2 + |a_1|^2 = 1$ ← "normalization"

Bit: 0, 1. Qubit: $|0\rangle, |1\rangle$ "standard basis"

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ = "zero state" = $|0\rangle$ "ket notation"

$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ = "one state" = $|1\rangle$

Common to write

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

"Superposition" = linear combination

" $|+\rangle$ is in a superposition of $|0\rangle$ and $|1\rangle$ "

Note: $|+\rangle, |-\rangle$

orthonormal basis

So $|0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$

" $|0\rangle$ is in a superposition of $|+\rangle$ and $|-\rangle$ "

Whether a state is a superposition or not depends on your basis. However, when we say superposition, we usually mean relative to standard basis

Bras & Kets

Linear algebra:

$$\vec{x} = \begin{pmatrix} x_0 \\ x_1 \\ \vdots \\ x_k \end{pmatrix}$$

conjugate transpose

$$\vec{x}^\dagger = (x_0^*, x_1^*, \dots, x_k^*)$$

complex conjugate

Quantum Computing:

$$|\psi\rangle = \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$$

"ket psi"

$$\langle\psi| = (a_0^* \ a_1^*)$$

"bra psi"

Useful fact

$$|\psi\rangle = a|\phi_0\rangle + b|\phi_1\rangle$$

$$(a, b \in \mathbb{C}, |\phi_0\rangle, |\phi_1\rangle \in \mathbb{C}^2)$$

$$\langle\psi| = a^*\langle\phi_0| + b^*\langle\phi_1|$$