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Probabilistic Computation

Deterministic Bit: $X=0$ or $X=1$

Probabilistic Bit: $P(0) = .75$ $P(1) = .25$

(↑ Probability of 0; ↑ Probability of 1)

Store info about probabilities in vectors

1-bit:

$$\begin{pmatrix} P(0) \\ P(1) \end{pmatrix}_A = \begin{pmatrix} .75 \\ .25 \end{pmatrix}$$

Another Bit

$$\begin{pmatrix} P(0) \\ P(1) \end{pmatrix}_B = \begin{pmatrix} .5 \\ .5 \end{pmatrix}$$

Together

$$\begin{pmatrix} .75 \\ .25 \end{pmatrix}_A \otimes \begin{pmatrix} .5 \\ .5 \end{pmatrix}_B = \begin{pmatrix} \frac{3}{8} \\ \frac{3}{8} \\ \frac{1}{8} \\ \frac{1}{8} \end{pmatrix} = \begin{pmatrix} P(00) \\ P(01) \\ P(10) \\ P(11) \end{pmatrix}$$

Classical Correlation:

$$\frac{1}{2}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

(Both bits have same value: $P(00) = P(11) = \frac{1}{2}$)

↑ can't be formed by combination of 2 independent bits

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Probabilistic n-bit State

$$\sum_{i \in \{0,1\}^n} a_i |i\rangle \quad a_i \geq 0$$

← not q. state, just vector

$$\sum a_i = 1$$

Probability of string i is a_i

Transform using left stochastic matrix

(preserves positivity & normalization)

Quantum n-bit State

$$\sum_{i \in \{0,1\}^n} a_i |i\rangle \quad a_i \in \mathbb{C}$$

$$\sum |a_i|^2 = 1$$

If measure in standard basis, Probability of outcome i is $|a_i|^2$

Transform using unitary matrix U: $U^\dagger U = UU^\dagger = I$

(preserves normalization)

Probabilistic Gate

Left Stochastic Matrix (columns sum to 1, non-negative entries)

↑ These matrices

ex: $\begin{pmatrix} 1/2 & 0 \\ 1/2 & 1 \end{pmatrix} \begin{pmatrix} p(0) \\ p(1) \end{pmatrix} = \begin{pmatrix} 1/2\, p(0) \\ 1/2\, p(0) + p(1) \end{pmatrix}$

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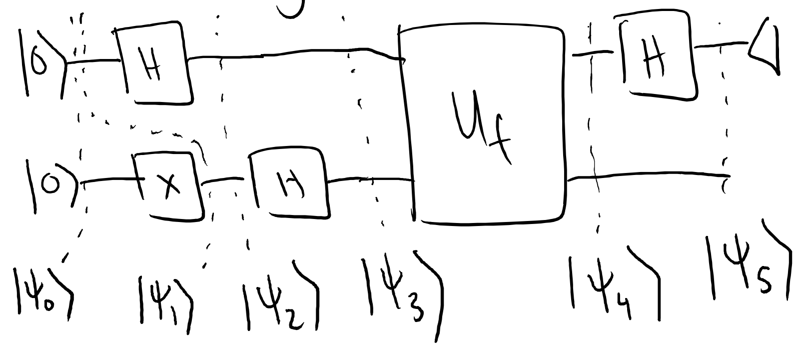
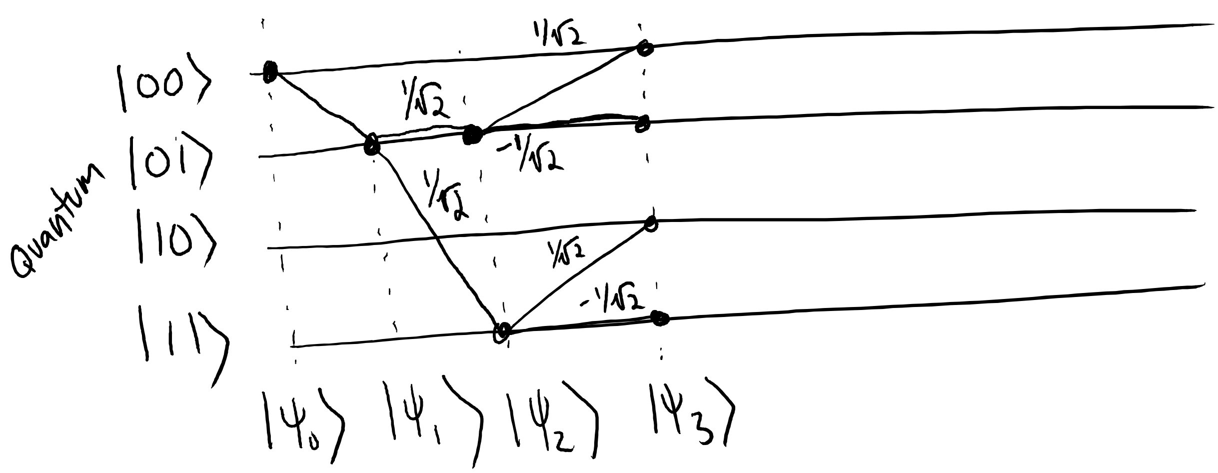
Deutsch Algorithm

$f(0) = f(1) = 1$



Compare Paths of Quantum vs. Probabilistic



(Treat H as coin flip)