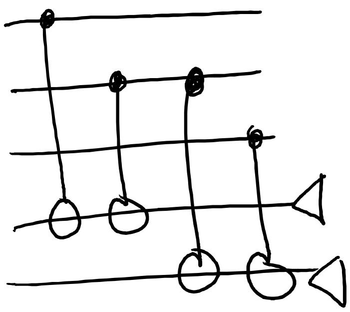
S.KIMMEL

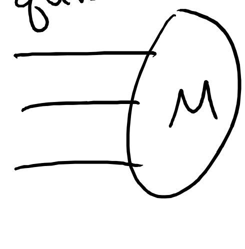
# New Math To Describe Partial Measurement

Partial on 5 qubits



is equivalent to

Effective on 3 qubits




**New measurement formalism:**

$M = \{P_i\}$

orthonormal states $\{|\phi_1\rangle, |\phi_2\rangle, \ldots\}$

$$P_i = \sum_{k \in S_i} |\phi_k\rangle\langle\phi_k|$$

← This type of matrix is called a <u>projector</u>.

$$\sum P_i = I$$

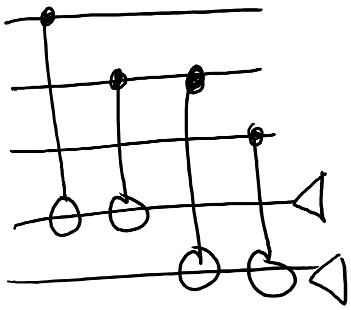
(identity matrix) (This means each k appears in exactly 1 set $S_i$.)

$$P_i P_j = \begin{cases} 0 & i \neq j \\ P_i & i = j \end{cases}$$

- Probability of Outcome $i$ : $\langle\psi|P_i|\psi\rangle$
- Collapse: $|\psi\rangle \rightarrow \dfrac{P_i|\psi\rangle}{\sqrt{\langle\psi|P_i|\psi\rangle}}$ ← normalization factor

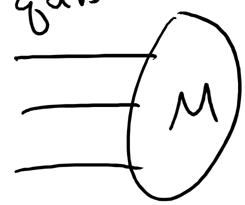
S. KIMMEL

Partial on 5 qubits



is equivalent to

Effective on 3 qubits




$$M = \left\{ \begin{array}{ll} P_0 & |000\rangle\langle 000| + |111\rangle\langle 111|, \\ P_1 & |100\rangle\langle 100| + |011\rangle\langle 011|, \\ P_2 & |010\rangle\langle 010| + |101\rangle\langle 101|, \\ P_3 & |001\rangle\langle 001| + |110\rangle\langle 110| \end{array} \right\}$$

$P_0 \longleftrightarrow$ Outcome $|00\rangle$ on ancilla

$P_1 \longleftrightarrow$ " $|10\rangle$

$P_2 \longleftrightarrow$ " $|11\rangle$

$P_3 \longleftrightarrow$ " $|01\rangle$

- Only 4 outcomes
- 8-Dim space

} does not fully collapse

S.KIMMEL

Q: If measure $a|000\rangle + b|011\rangle + c|100\rangle$ with M, which outcomes are possible?

A) $P_0, P_1$

B) $P_1, P_2$

C) $P_0, P_2$

D) $P_1, P_3$

Q: If get outcome $P_1$ when measure $a|000\rangle + b|011\rangle + c|100\rangle$, what does state collapse to?

A) $b|011\rangle + c|100\rangle$

B) $(b|011\rangle + c|100\rangle)\frac{1}{\sqrt{|b|^2+|c|^2}}$

c) $b|011\rangle$

d) $c|100$

S.KIMMEL

Q: If measure $a|000\rangle + b|011\rangle + c|100\rangle$ with $M$, which outcomes are possible?

A) $P_0, P_1$ $\Leftarrow$ all others $P_i(a|000\rangle + b|011\rangle + c|100\rangle) = 0$

B) $P_1, P_2$

C) $P_0, P_2$

D) $P_1, P_3$

Q: If get outcome $P_1$ when measure $a|000\rangle + b|011\rangle + c|100\rangle$, what does state collapse to?

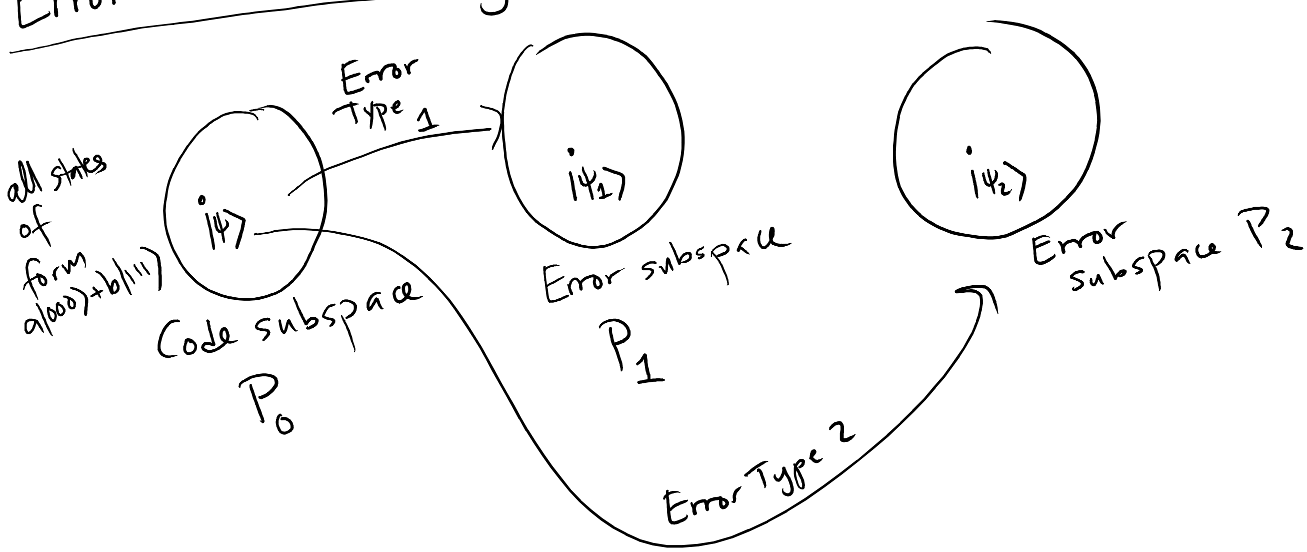
A) $b|011\rangle + c|100\rangle$

B) $(b|011\rangle + c|100\rangle)\frac{1}{\sqrt{|b|^2+|c|^2}}$ $\Leftarrow$ $\frac{P_1|\psi\rangle}{\sqrt{\langle\psi|P_1|\psi\rangle}}$

c) $b|011\rangle$

d) $c|100$

## Error Correction Big Idea




Measurement doesn't cause full collapse, just tells you type of error. Doesn't tell you about a, b.

S.KIMMEL

## Continuous Errors

Suppose $|\psi\rangle \in$ code

Error Type 1: $|\psi\rangle \to |\psi_1\rangle$

Error Type 2: $|\psi\rangle \to |\psi_2\rangle$

What if get combination of error 1 and 2 and get superposition:

$$|\psi\rangle \to \alpha_0|\psi\rangle + \alpha_1|\psi_1\rangle + \alpha_2|\psi_2\rangle$$

When measure using $\{P_0, P_1, P_2\}$, Collapsed state?

- $P_0|\psi\rangle = |\psi\rangle$

  Outcome $P_0 \Rightarrow$ collapse to $|\psi\rangle$ ✓

- $P_1|\psi\rangle \to |\psi_1\rangle$

  Outcome $P_1 \Rightarrow$ collapse to $|\psi_1\rangle \to$ do correction ✓

- $P_2|\psi\rangle = |\psi_2\rangle$

  Outcome $P_2 \to$ collapse to $|\psi_2\rangle \to$ do correction

Measurement forces system to choose whether a full error occured or not. Even though infinite possible Bloch sphere rotations, after measurement, collapses to states corresponding to only a couple of allowed errors.

S.KIMMEL

$X|0\rangle = |1\rangle \quad X|1\rangle = |0\rangle$ } We have a way to fix

$Z|+\rangle = |-\rangle \quad Z|-\rangle = |+\rangle$ ← Same relationship!

Replace all $|0\rangle$ with $|+\rangle$ ⟹ get way to fix Z

" $|1\rangle$ with $|-\rangle$

$c|+\rangle + d|-\rangle \rightarrow c|{+}{+}{+}\rangle + d|{-}{-}{-}\rangle$

$M = \{ |{+}{+}{+}\rangle\langle{+}{+}{+}| + |{-}{-}{-}\rangle\langle{-}{-}{-}|, \ \dots \}$

## Shor 9-qubit Code:

$|0_L\rangle$ ← logical qubit

Concatenate: $|+\rangle \rightarrow |{+}{+}{+}\rangle \xrightarrow{|0\rangle \rightarrow |000\rangle} (|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle) = |0_L\rangle$

$|-\rangle \rightarrow |{-}{-}{-}\rangle \xrightarrow[|1\rangle \rightarrow |111\rangle]{} (|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle) = |1_L\rangle$

9 physical qubits

## Measurement to detect errors

($X_1$ = X acting on first qubit)

$$P_0 = |0_L\rangle\langle 0_L| + |1_L\rangle\langle 1_L|$$

$$P_{X1} = X_1|0_L\rangle\langle 0_L| + X_1|1_L\rangle\langle 1_L|$$

$$P_{Z1} = Z_1|0_L\rangle\langle 0_L| + Z_1|1_L\rangle\langle 1_L|$$

$$P_{Y1} = Y_1|0_L\rangle\langle 0_L| + Y_1|1_L\rangle\langle 1_L|$$

Orthogonal!

$P_0 P_{Y1} = P_{X1} P_{Z1} = P_{Y1} P_{Z1} = 0$

S.KIMMEL

Q: How do you tell if error $Z_1$ or $Z_2$ occured?

A) Whether you project onto $P_{Z1}$ or $P_{Z2}$

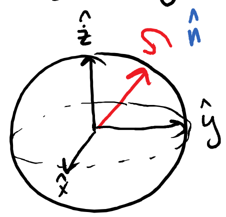
B) You can't distinguish these errors, so the code fails

C) You can't distinguish these errors, but it doesn't matter

$$Z_1|0_L\rangle = (|000\rangle - |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle) = Z_2|0_L\rangle$$
$$Z_1|1_L\rangle = (|000\rangle + |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle) = Z_2|1_L\rangle$$

- Projector detects if a $Z$ error occurs on <u>any</u> qubit in 1st block
- Correction is same in each case! Apply $Z$ to <u>any</u> qubit in first block.

Single qubit unitary




$$U = \cos\frac{\theta}{2} I - i \sin\frac{\theta}{2}\left(n_x X + n_y Y + n_z Z\right)$$

$$= aI + bX + cY + dZ$$

$$\begin{aligned} U\left(x|0_L\rangle + y|1_L\rangle\right) = \; & a\left(x|0_L\rangle + y|1_L\rangle\right) \\ & + b\left(x X|0_L\rangle + y X|1_L\rangle\right) \\ & + c\left(x Y|0_L\rangle + y Y|1_L\rangle\right) \\ & + d\left(x Z|0_L\rangle + y Z|1_L\rangle\right) \end{aligned}$$

When do projective measurement, collapses to just one of these outcomes. Then fix as needed.

S.KIMMEL

# Projectors in Shor's 9-qubit code

$|0_L\rangle = (|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)$

$|1_L\rangle = (|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)$

$P_0 = |0_L\rangle\langle 0_L| + |1_L\rangle\langle 1_L|$

$P_{X_1} = X_1|0_L\rangle\langle 0_L|X_1 + X_1|1_L\rangle\langle 1_L|X_1$

$X_1|0_L\rangle = (|100\rangle + |011\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)$

$X_1|1_L\rangle \; (|100\rangle - |011\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)$

Key: $P_0 P_{X_1} = 0$ (b/c $\langle 0_L|(X|0_L\rangle = 0$
$\langle 0_L| X|1_L\rangle = 0$
$\langle 1_L|X|0_L\rangle = 0$ )

So $X_1$ takes element of code to orthogonal subspace (different "bubble")

Q: Which of the following errors can be accurately corrected by Shor's 9-qubit code? Code corrects by assuming fewest number of errors possible occured

A) $Z_1 Z_2$

B) $Z_1 Z_4$

C) $Z_1 Z_2 Z_3$

D) $X_1 Z_2$

E) $X_1 X_2$

F) $X_1 X_4$

G) $Y_1 Z_4$

All these errors take code to ortho. subspace. Issue is whether fewer # of errors could do same thing

S.KIMMEL

✓ A) $Z_1 Z_2$

✗ B) $Z_1 Z_4$

✓ C) $Z_1 Z_2 Z_3$

✓ D) $X_1 Z_2$

✗ E) $X_1 X_2$

✓ F) $X_1 X_4$

✗ G) $Y_1 Z_4$

# Issues

## Threshold

- Shor Code: Corrects 1 error
- Need to complete correction circuit before $2^{nd}$ error occurs
- Current error rates too high. Need to be below some critical rate: threshold $\boxed{\sim 10^{-4} \text{ prob of error}}$

getting close

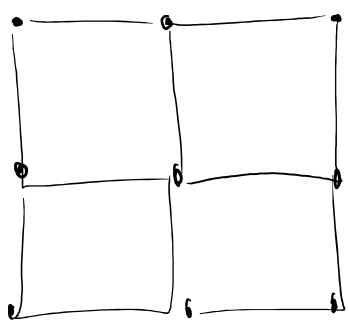
## Gates

- Need to apply gates to logical qubits
- If error occurs, gate implementation circuit can make error spread.
- H, CNOT = OK, $\boxed{T = \text{NOT OK}}$ ← we have ways to deal with this, but not great

S.KIMMEL

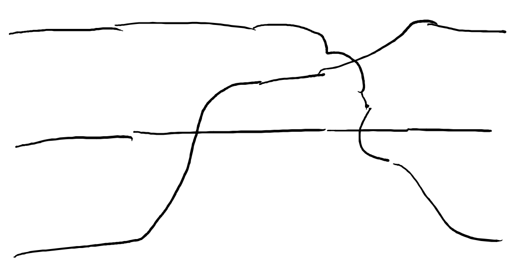
# Current Research in Codes

- Flag qubits:
  - 19 physical qubits (including ancillas)
  - 7 logical qubits
  - Protects against 1 qubit errors
- Color Codes



- qubits usually on a grid
- easier to apply CNOT on edges
- color codes are easier to implement respecting this locality

- Topological codes (* I don't buy)



- gates implemented by braiding
- errors only result from braids → random errors unlikely to braid if strands kept far apart
- Need physical systems that might