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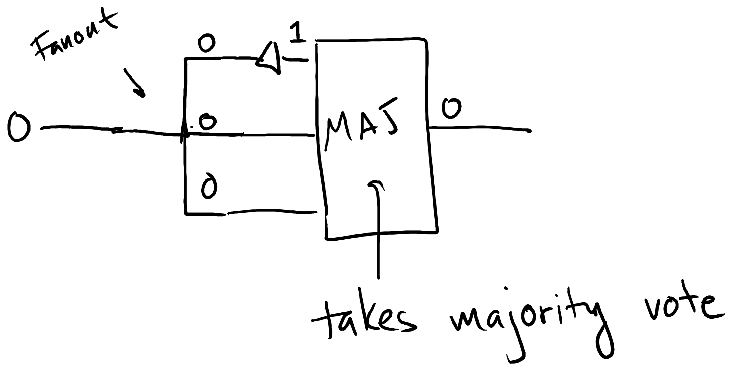
Errors

- Classical computers:
  - Cosmic Rays are largest source
  - 1 bit flip per 4GB per day
  - $0 \to 1$ or $1 \to 0$

Usually just don't worry about it... (although this can backfire)

Solution: Repetition Code



- Quantum
  - laser shines on 2 qubits instead of 1
  - Other atom bounces into your atom
  - Heat (anomalous heating)
  - Magnetic fields
  - Electric fields
  - laser shape not perfect
  - system is actually qudit, not qubit

Many error sources

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Q: Why won't repetition code work for qubits

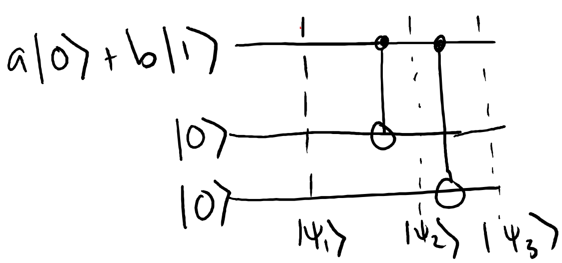
A) MAJ not reversible

B) We can have more varied errors in quantum case

C) No FANOUT because no cloning

D) All of the above

Instead, use this circuit



$|\psi_1\rangle = a|000\rangle + b|100\rangle$

$|\psi_2\rangle = a|000\rangle + b|110\rangle$

$|\psi_3\rangle = a|000\rangle + b|111\rangle$

Encode $a|0\rangle + b|1\rangle \longrightarrow a|000\rangle + b|111\rangle$

Similar to repetition code...

but more quantum-y!

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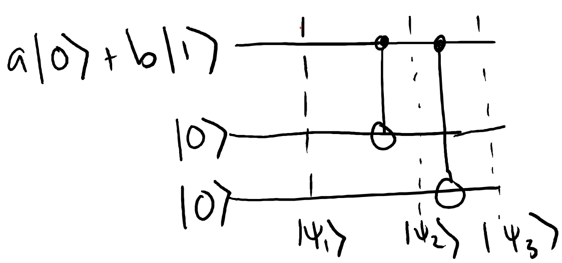

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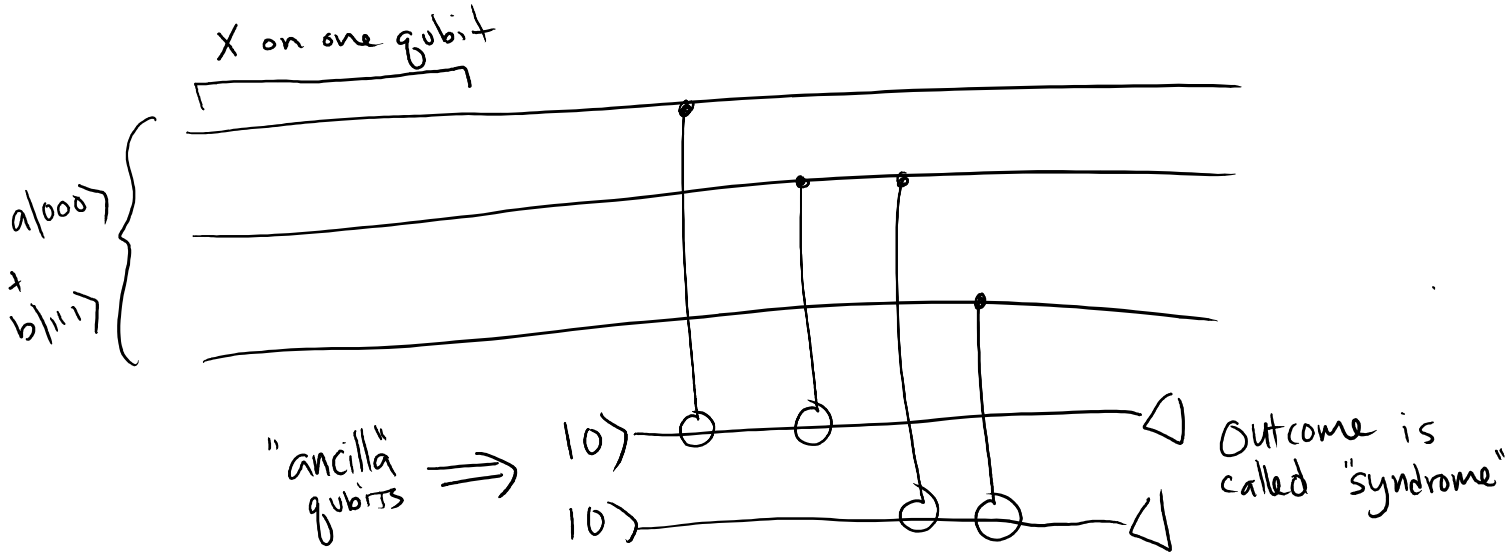

1. What measurement outcomes do you get with which probability?
2. What does the rest of the state collapse to?
3. How do you fix error based on outcome
4. What happens if Z error on a qubit?

	1.	2.	3.
• If $\boxed{X}$ on 1st,	Outcome $\vert 10\rangle$ →	$a\vert 100\rangle + b\vert 111\rangle$ →	X on 1
• If $\boxed{X}$ on 2nd,	outcome $\vert 11\rangle$ →	$a\vert 010\rangle + b\vert 101\rangle$ →	X on 2
• If $\boxed{X}$ on 3rd	outcome $\vert 01\rangle$ →	$a\vert 001\rangle + b\vert 001\rangle$ →	X on 3
• If no X,	outcome $\vert 00\rangle$ →	$a\vert 000\rangle + b\vert 111\rangle$ →	nothing

4. Get outcome $|00\rangle \Rightarrow$ no detection

Don't need to know a, b

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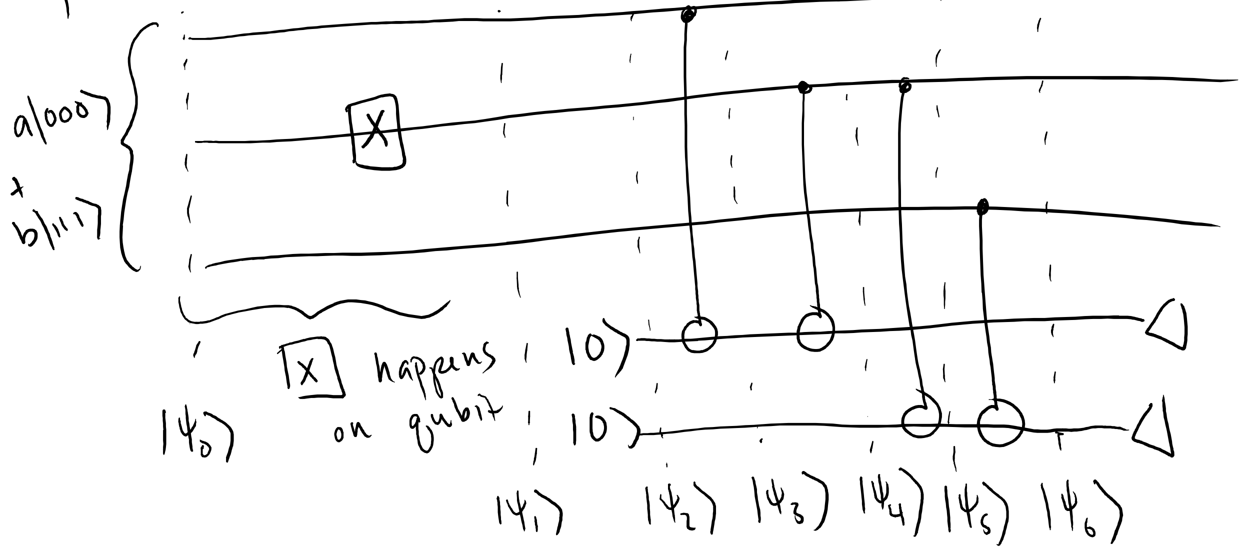
Sample calculation (X on second qubit)



$$|\psi_1\rangle = X_2\left(a|000\rangle + b|111\rangle\right) = a|010\rangle + b|101\rangle$$

$$|\psi_2\rangle = a|010\rangle|00\rangle + b|101\rangle|00\rangle$$

$$|\psi_3\rangle = a|010\rangle|00\rangle + b|101\rangle|10\rangle$$

$$|\psi_4\rangle = a|010\rangle|10\rangle + b|101\rangle|10\rangle$$

$$|\psi_5\rangle = a|010\rangle|11\rangle + b|101\rangle|10\rangle$$

$$|\psi_6\rangle = a|010\rangle|11\rangle + b|101\rangle|11\rangle$$

$$= \left(a|010\rangle + b|101\rangle\right)|11\rangle$$

Apply X $\rightarrow$ $a|000\rangle + b|111\rangle$

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New mathematical description of partial measurement

(Mathematically equivalent to appending ancillas, interacting, measure ancillas)

Old: $M = \{|\phi_i\rangle\}$ orthornormal basis Outcome i with prob $|\langle\psi|\phi_i\rangle|^2$

$|\psi\rangle \to |\phi_i\rangle$

New: $M = \{P_i\}$

$$P_i = \sum_{k \in S_i} |\phi_k\rangle\langle\phi_k|$$ ← orthonormal states (P_i are "projectors")

$\sum P_i = \mathbb{I}$ ($|\phi_k\rangle\langle\phi_k|$ appears in exactly 1 projector)

$$P_i P_j = \begin{cases} 0 & i \neq j \\ P_i & i = j \end{cases}$$

Outcome $i \to \Pr(i) = \langle\psi|P_i|\psi\rangle$

$$|\psi\rangle \to \frac{P_i|\psi\rangle}{\sqrt{\langle\psi|P_i|\psi\rangle}}$$

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Measurement for Error Correction of X:

$$M = \{ \underbrace{|000\rangle\langle 000| + |111\rangle\langle 111|}_{P_0},\ \underbrace{|100\rangle\langle 100| + |011\rangle\langle 011|}_{P_1},\ \underbrace{|010\rangle\langle 010| + |101\rangle\langle 101|}_{P_2},\ \underbrace{|001\rangle\langle 001| + |110\rangle\langle 110|}_{P_3} \}$$

$P_0 \longleftrightarrow$ Outcome $|00\rangle$ on ancilla

$P_1 \longleftrightarrow$ " $|10\rangle$

$P_2 \longleftrightarrow$ " $|11\rangle$

$P_3 \longleftrightarrow$ " $|01\rangle$

- Only 4 outcomes
- 8-Dim space

$\Big\}$ does not fully collapse

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Q: If measure $a|000\rangle + b|011\rangle + c|100\rangle$ with M, which outcomes are possible?

A) P_0, P_1

B) P_1, P_2

C) P_0, P_2

D) P_1, P_3

Q: If get outcome P_1 when measure $a|000\rangle + b|011\rangle + c|100\rangle$, what does state collapse to?

A) $b|011\rangle + c|100\rangle$

B) $(b|011\rangle + c|100\rangle)\frac{1}{\sqrt{|b|^2+|c|^2}}$

c) $b|011\rangle$

d) $c|100$

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Q: If measure $a|000\rangle + b|011\rangle + c|100\rangle$ with M, which outcomes are possible?

A) P_0, P_1 $\Leftarrow$ all others $P_i\left(a|000\rangle + b|011\rangle + c|100\rangle\right) = 0$

B) P_1, P_2

C) P_0, P_2

D) P_1, P_3

Q: If get outcome P_1 when measure $a|000\rangle + b|011\rangle + c|100\rangle$, what does state collapse to?

A) $b|011\rangle + c|100\rangle$

B) $\left(b|011\rangle + c|100\rangle\right)\frac{1}{\sqrt{|b|^2+|c|^2}}$ $\Leftarrow$ $\frac{P_1|\psi\rangle}{\sqrt{\langle\psi|P_1|\psi\rangle}}$

c) $b|011\rangle$

d) $c|100$