S. Kimmel

Basic Quantum Algorithms

Goal:

- Strategies for analyzing quantum algorithms
- Understanding of why quantum algs. can do better than quantum.

Deutsch Algorithm

Consider a one bit function f:

x	$f(x)$
0	?
1	?

Options for f:

x	$f(x)$
0	0
1	0

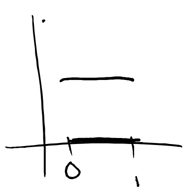
x	$f(x)$
X	1
1	1

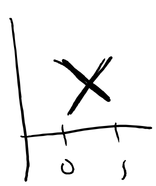
"even"

x	$f(x)$
0	0
1	1

x	$f(x)$
0	1
1	0

"balanced"





Problem: Decide if f is even or balanced.

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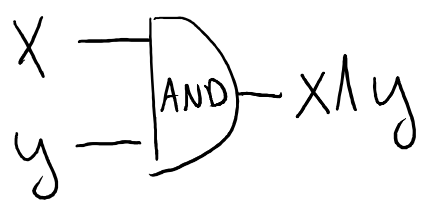
How is f given?

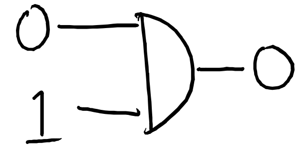
- Classically: $x - \boxed{f} - f(x)$

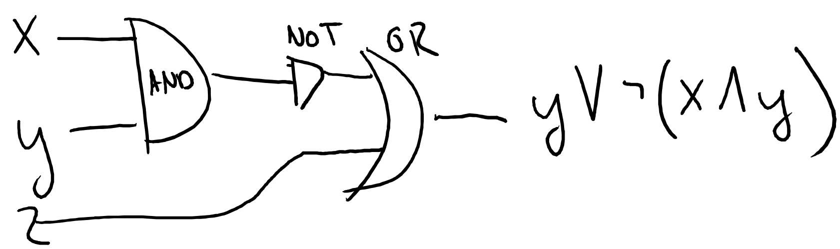
$0 - \boxed{f} - f(0)$

$1 - \boxed{f} - f(1)$

Circuit model of Classical Computation







Q: Can we create a unitary version?

U_f:

$|0\rangle \longrightarrow |f(0)\rangle$

$|1\rangle \longrightarrow |f(1)\rangle$

ex: $f(0) = 1$
$f(1) = 0$

$U_f|0\rangle = |1\rangle$
$U_f|1\rangle = |0\rangle$

A. This is a good unitary

B. This is not a good unitary because we haven't defined its effect on $|+\rangle$

C. This is not a good unitary because it depends on f.

D. This is not a good unitary because it is not always reversible.

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D. This is not a good unitary because it is not always reversible.

ex: $f(0)=1 \quad f(1)=1$

$U_f|0\rangle = |1\rangle$
$U_f|1\rangle = |1\rangle$

$$U_f^\dagger|1\rangle \to |0\rangle \text{ or } |1\rangle$$

Reverses U_f

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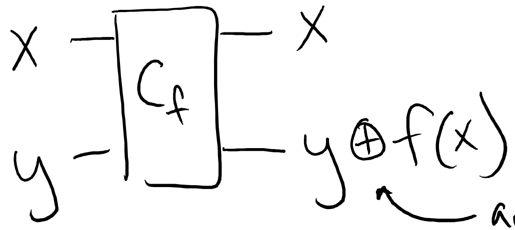
How do we make f reversible classically

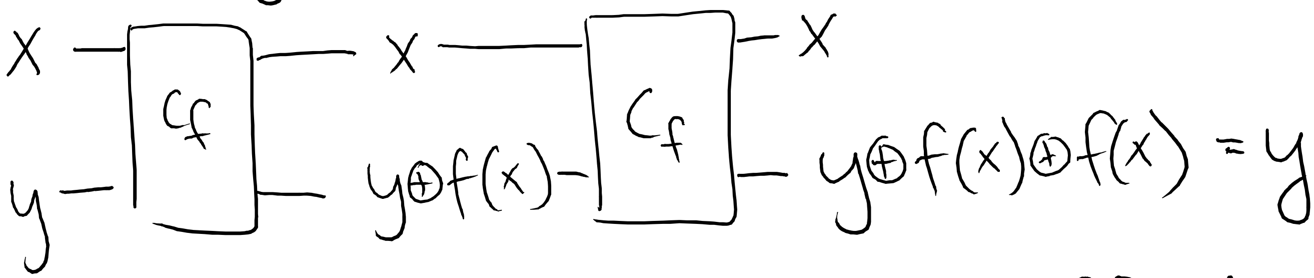
- Classically: $x - \boxed{f} - f(x)$ (non reversible)



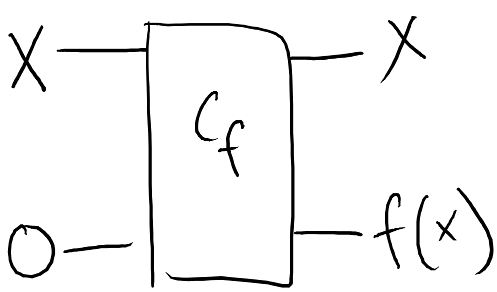
(reversible)

addition mod 2

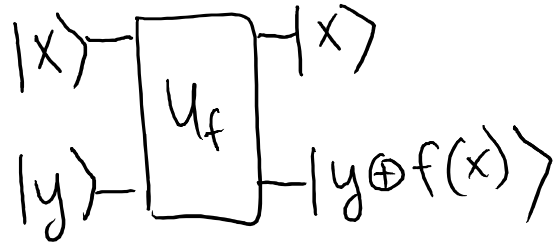
Self-Inverting



- $f(x)=0 \Rightarrow y \oplus 0 \oplus 0 = y$
- $f(x)=1 \Rightarrow y \oplus 1 \oplus 1 = y$



Reversible Quantum f : just add kets!



ex: $f(0)=1$, $f(1)=0$

$$\left.\begin{aligned}
U_f|0\rangle|0\rangle &= |0\rangle|0\oplus f(0)\rangle = |0\rangle|1\rangle \\
U_f|1\rangle|1\rangle &= |1\rangle|1\oplus f(1)\rangle = |1\rangle|1\rangle \\
U_f|0\rangle|1\rangle &= |0\rangle|1\oplus f(0)\rangle = |0\rangle|0\rangle \\
U_f|1\rangle|0\rangle &= |1\rangle|0\oplus f(1)\rangle = |1\rangle|0\rangle
\end{aligned}\right\} \text{Can check it is reversible (self inverting)}$$

Once we define how U_f acts on standard basis, we've defined unitary

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Problem (precise): How many uses of C_f / U_f are required by a classical/quantum circuit to determine if f is balanced or even?

classical answer

quantum answer

Classical Query Complexity

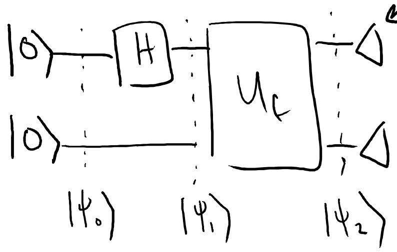
Quantum Query Complexity

Classical query complexity of even/odd = 2

Why: If learn $f(0)$ only, not enough to decide if even or balanced

If learn $f(1)$ only, not enough to decide if even or balanced

Using Superposition



☆ H creates superposition

☆ Allows you to ask value of $f(0)$ and $f(1)$ at the same time

$$|\psi_0\rangle = |00\rangle$$

$$|\psi_1\rangle = H|0\rangle \otimes I|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \otimes |0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle|0\rangle + |1\rangle|0\rangle\right)$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}\left(U_f|00\rangle + U_f|10\rangle\right) = \frac{1}{\sqrt{2}}\left(|0\rangle|0 \oplus f(0)\rangle + |1\rangle|0 \oplus f(1)\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(|0\rangle|f(0)\rangle + |1\rangle|f(1)\rangle\right)$$

standard basis states

(If $f(0)=1, f(1)=0$ this is the state $\frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle$)

Seems great! We are in superposition of two function values! Classically, if apply C_f once, only get info about one value of function

Now measure in standard basis: $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$

- With prob $1/2$ get outcome $|0\rangle|f(0)\rangle$
- With prob $1/2$ get outcome $|1\rangle|f(1)\rangle$

☆ Superposition is not enough to give (usually) a quantum speed-up. Need something more

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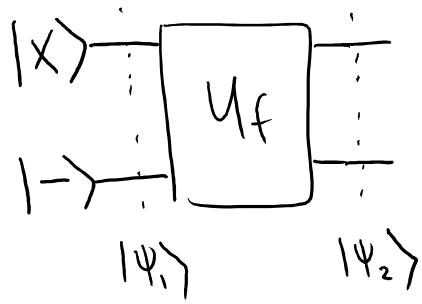
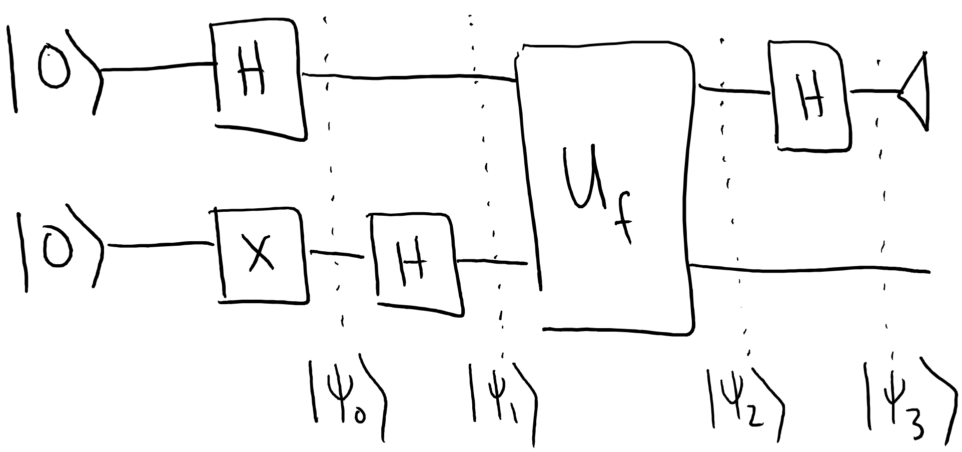
Analyze the following circuits.

$|x\rangle = |0\rangle$ or $|1\rangle$



Explain why $|\psi_2\rangle = (-1)^{f(x)}|x\rangle|-\rangle$



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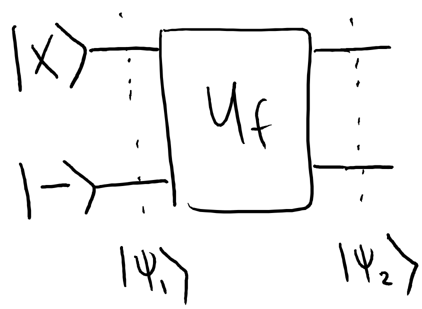

Explain why $|\psi_2\rangle = (-1)^{f(x)}|x\rangle|-\rangle$

$$|\psi_2\rangle = U_f|x\rangle|-\rangle = U_f \frac{1}{\sqrt{2}}\left(|x\rangle|0\rangle - |x\rangle|1\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(|x\rangle|f(x)\rangle - |x\rangle|f(x)\oplus 1\rangle\right)$$

$f(x) = 0$: $\frac{1}{\sqrt{2}}|x\rangle|+\rangle$

$f(x) = 1$: $-\frac{1}{\sqrt{2}}|x\rangle|+\rangle$

Phase kickback:

Information about $f(x)$ is stored in phase ($+1$ or -1) instead of in $|0\rangle$ or $|1\rangle$ state.

"Kickback"... will explain in next circuit