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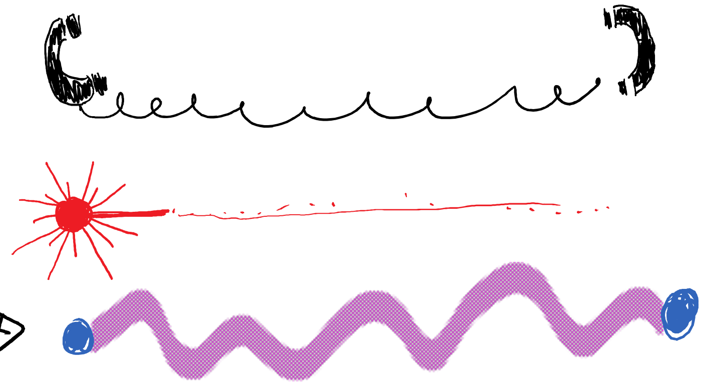
Teleportation

Alice Bob

Resources



"cbit" = 1 bit classical info

"qbit" = 1 qubit

"ebit" = 2 qubits in entangled state

ebit gets "used up" by being measured or being sent to Bob

$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

Question: If Alice wants to send something to Bob, what resources does she need?

- How many qubits need to be sent to communicate 1 cbit?

  A) 0 B) 1 C) 2 D) ∞

- How many ebits are required to communicate 1 cbit?

  A) 0 B) 1 C) 2 D) ∞ is not enough

- How many cbits are needed to communicate 1 qbit?

  A) 1 B) 2 C) log in precision D) ∞ is not enough

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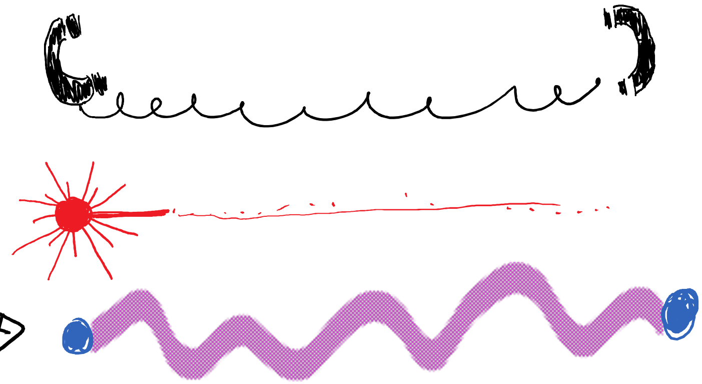

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$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

Question: If Alice wants to send something to Bob, what resources does she need?

Task	Resources Required
Send 1 cbit	• 1 cbit or • 1 qbit (A sends $|0\rangle$ or $|1\rangle$, B measures $\{|0\rangle, |1\rangle\}$) or • ~~1 ebit~~ can't communicate with ebit
Send 2 cbits	• 2 cbit • 2 qbit • 1 ebit + 1 qubit "Superdense coding"

don't have time to discuss. Can read about

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Task	Resources	
Send 1 qbit	• _ cbits	(need to describe 2 real #'s of Bloch sphere)
	• 1 qbits	"
	• 1 ebit + 2 cbits	

Strategy

1. A & B start with $|\psi\rangle_{A_1} |\beta_{00}\rangle_{A_2 B}$ (state to send: $|\psi\rangle$; ebit shared: $|\beta_{00}\rangle$) (1 ebit)
2. Alice measures A_1 and A_2 (this destroys entanglement)
3. Alice sends outcome of measurement to Bob (2 cbits)
4. Bob applies a unitary to his system B based on Alice's cbits

New skill: what happens when only part of system is measured?

Recall: If both measured, effective measurement is

$$M_A \otimes M_B$$

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Partial Measurement

Let $|\psi\rangle_{AB}$ be a state on systems A (N_A-dim) and B (N_B-dim)

- Alice measures with $M_A = \{|\phi_1\rangle, \ldots |\phi_{N_A}\rangle\}$,
- Bob does not measure → If Alice gets outcome $|\phi_i\rangle$, what happens to Bob's state?

1. It is always possible to write (uniquely)

$$|\psi_{AB}\rangle = \sum_{i=0}^{N_A} a_i |\phi_i\rangle_A |\chi_i\rangle_B$$

Complex number, where $\sum |a_i|^2 = 1$

element of M_A

N_B dimensional, normalized state

2. Then Alice gets outcome $|\phi_i\rangle$ with probability $|a_i|^2$, and Bob's system collapses to:

$$|\phi_i\rangle_A |\chi_i\rangle_B$$

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How to change basis

Orthonormal basis $\{|\psi_i\rangle\}$

$$\mathbb{I} = \sum_i |\psi_i\rangle\langle\psi_i|$$

identity matrix

$$\begin{pmatrix} a \\ b \end{pmatrix}\begin{pmatrix} a^* & b^* \end{pmatrix} = \begin{pmatrix} aa^* & ab^* \\ a^*b & bb^* \end{pmatrix}$$

Ketbra $\Longrightarrow$ matrix

Write $|\phi\rangle$ using $\{|\psi_i\rangle\}$ basis:

amplitude

basis state

$$|\phi\rangle = \mathbb{I}\,|\phi\rangle = \sum_i |\psi_i\rangle\langle\psi_i|\,|\phi\rangle = \sum_i \left(\langle\psi_i|\phi\rangle\right)|\psi_i\rangle$$

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ex:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle_{AB} + |11\rangle_{AB}\right)$$

Measure A in $\{|+\rangle, |-\rangle\}$
- What outcomes occur with what probability?
- What happens to B system?

1. Put system A into $\{|+\rangle, |-\rangle\}$ basis:

$$I_{AB} = I_A \otimes I_B = \left(|+\rangle\langle+| + |-\rangle\langle-|\right) \otimes I_B$$

$$I_{AB}|\psi\rangle = \left(|+\rangle\langle+| + |-\rangle\langle-|\right)_A \otimes I_B \left(\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)\right)_{AB}$$

$$= \frac{1}{\sqrt{2}}\left(|+\rangle\underbrace{\langle+|0\rangle_A}|0\rangle_B + |+\rangle\underbrace{\langle+|1\rangle_A}|1\rangle_B + |-\rangle\underbrace{\langle-|0\rangle_A}|0\rangle_B + |-\rangle\underbrace{\langle-|1\rangle_A}|1\rangle_B\right)$$

$$= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|+\rangle|0\rangle + \frac{1}{\sqrt{2}}|+\rangle|1\rangle + \frac{1}{\sqrt{2}}|-\rangle|0\rangle + \frac{-1}{\sqrt{2}}|-\rangle|1\rangle\right)$$

$$= \frac{1}{2}\left(|+\rangle\left(|0\rangle + |1\rangle\right) + |-\rangle\left(|0\rangle - |1\rangle\right)\right)$$

not normalized

$$= \frac{1}{\sqrt{2}}\left(|+\rangle\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) + |-\rangle\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)\right)$$

$$= \frac{1}{\sqrt{2}}\left(|+\rangle|+\rangle + |-\rangle|-\rangle\right)$$

$|+\rangle$: $\Pr\left(\frac{1}{2}\right)$ $\quad B \to |+\rangle$

$|-\rangle$: $\Pr\left(\frac{1}{2}\right)$ $\quad B \to |-\rangle$