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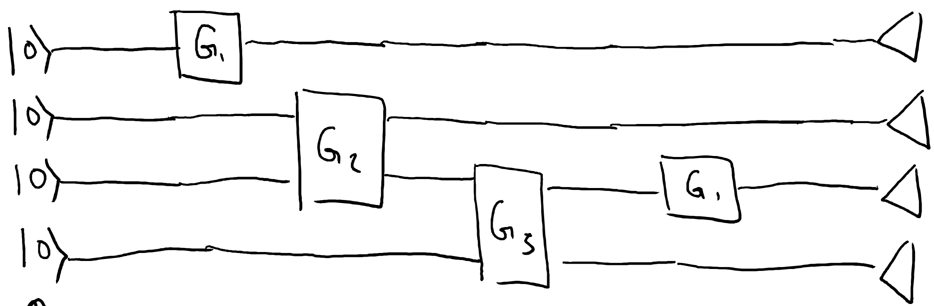
Quantum Circuits

Standard Q. Circuit Model



Measure each qubit in $\{|0\rangle, |1\rangle\}$ basis

$\Downarrow$

results in global standard basis measurement

Start $|0\rangle_{top} \otimes |0\rangle \cdots \otimes |0\rangle_{bottom}$

$$\underbrace{|0\rangle|0\rangle|0\rangle\cdots|0\rangle}_{n \text{ qubits}} = |0\rangle^{\otimes n}$$

- In classical computing, all circuits built from AND, NOT (universal)
- In quantum, all circuits built from

H, T, CNOT (universal)

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

Using these 3 gates can get arbitrarily close to any unitary

Sometimes use other pre-compiled gates: X, Z, etc. in our circuits for ease of understanding

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Multi-Qubit Gates

- Quantum Control Gate:
  - 2-Qubit Controlled Not "CNOT"

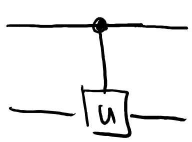
"controlled on first qubit, apply X to second qubit"

Acts on standard basis states like CNOT

$$
\left.
\begin{aligned}
|00\rangle &\to |00\rangle \\
|01\rangle &\to |01\rangle \\
|10\rangle &\to |1\rangle X|0\rangle = |1\rangle|1\rangle = |11\rangle \\
|11\rangle &\to |1\rangle X|1\rangle = |1\rangle|0\rangle = |10\rangle
\end{aligned}
\right\}
\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}
$$

CNOT: $|0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X$

More generally, controlled U operation

- $C\text{-}U =$ 

$$|0\rangle|i\rangle \to |0\rangle|i\rangle \qquad : |0\rangle\langle 0| \otimes I$$
$$|1\rangle|i\rangle \to |1\rangle U|i\rangle \qquad + |1\rangle\langle 1| \otimes U$$