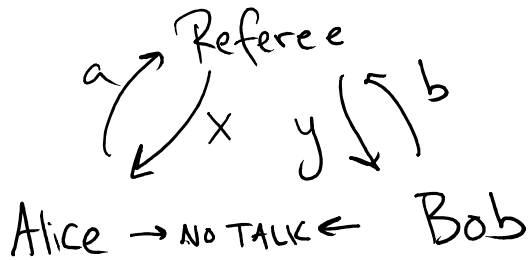


Goals

- Calculate 2-qubit measurement outcomes
- Describe quantum gates
- Analyze bomb detection

(away next Thurs)

## CHSH



$$|\psi\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$

Winning Condition:  
 $x \neq y = a \oplus b$

Can enforce no communication using speed of light.

$$M(\theta) = \left\{ \begin{array}{l} \cos\theta |0\rangle + \sin\theta |1\rangle, \\ -\sin\theta |0\rangle + \cos\theta |1\rangle \end{array} \right\}$$

$\uparrow$   $|\phi_0(\theta)\rangle$                        $\uparrow$   $|\phi_1(\theta)\rangle$

x	Alice Measures
0	$M(0) = \{ 0\rangle,  1\rangle\}$
1	$M(\frac{\pi}{4}) = \{ +\rangle,  -\rangle\}$

y	Bob Measures
0	$M(\frac{\pi}{8})$
1	$M(-\frac{\pi}{8})$

Outcome  $|\phi_0(\theta)\rangle \rightarrow$  Respond "0"

Outcome  $|\phi_1(\theta)\rangle \rightarrow$  Respond "1"

Case 1

$$x=y=0 \quad x \neq y=0 \rightarrow a \oplus b = 0$$

$$a=b=0 \quad a=b=1$$

$$M = \left\{ \begin{array}{l} |\phi_0(0)\rangle |\phi_0(\frac{\pi}{8})\rangle, |\phi_0(0)\rangle |\phi_1(\frac{\pi}{8})\rangle, |\phi_1(0)\rangle |\phi_0(\frac{\pi}{8})\rangle \\ |\phi_1(0)\rangle |\phi_1(\frac{\pi}{8})\rangle \end{array} \right\}$$

Good outcomes  
( $a=b=0$ )  
( $a=b=1$ )

$$P(a=b=0)$$

$$\begin{aligned} & \left| \langle \phi_0(0) | \langle \phi_0(\frac{\pi}{8}) | \frac{1}{\sqrt{2}} |00\rangle + \frac{1}{\sqrt{2}} |11\rangle \right|^2 \\ &= \left| \frac{1}{\sqrt{2}} \langle \phi_0(0) | 0 \rangle \langle \phi_0(\frac{\pi}{8}) | 0 \rangle + \frac{1}{\sqrt{2}} \langle \phi_0(0) | 1 \rangle \langle \phi_0(\frac{\pi}{8}) | 1 \rangle \right|^2 \\ &= \left| \frac{1}{\sqrt{2}} \cos(\frac{\pi}{8}) \right|^2 \end{aligned}$$

$$P(a=b=1)$$

$$\begin{aligned} &= \left| \frac{1}{\sqrt{2}} \langle \phi_1(0) | 0 \rangle \langle \phi_1(\frac{\pi}{8}) | 0 \rangle + \frac{1}{\sqrt{2}} \langle \phi_1(0) | 1 \rangle \langle \phi_1(\frac{\pi}{8}) | 1 \rangle \right|^2 \\ &= \left| \frac{1}{\sqrt{2}} \cos(\frac{\pi}{8}) \right|^2 \end{aligned}$$

$$\text{Total} \quad \frac{1}{2} \cos^2(\frac{\pi}{8}) + \frac{1}{2} \cos^2(\frac{\pi}{8}) = \cos^2(\frac{\pi}{8}) = .85$$

Get same for any value of  $x, y$ ! Beats .75!