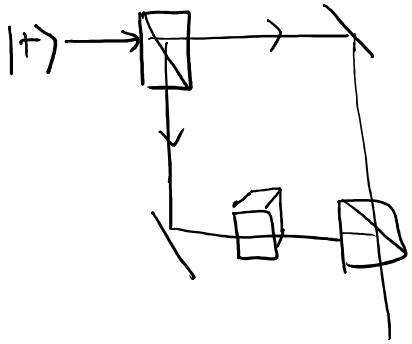
S. KIMMEL

Goals

- Analyze bomb detection
- Describe unitaries



Photon is in a superposition of multiple paths
positions
locations

Big Idea: Put bomb in one path. Less likely to explode, and maybe can learn something without exploding

---

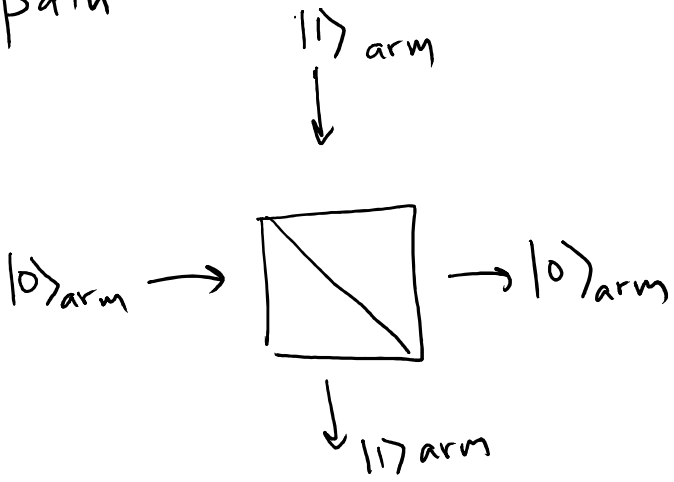
Mathematically, 1 photon described by polarization and path : 2 qubits!

$$|\psi\rangle_P |\phi\rangle_A$$

↑ polarization ↖ path (A for "arm")

S. KIMMEL

Beamsplitter is an operation that entangles polarization + path



How does beamsplitter affect standard basis:

$$|0\rangle_P|0\rangle_A \rightarrow |0\rangle_P|0\rangle_A \quad \Leftrightarrow \quad \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \rightarrow \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$

$$|0\rangle_P|1\rangle_A \rightarrow |0\rangle_P|1\rangle_A \quad \Leftrightarrow \quad \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \rightarrow \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

$$|1\rangle_P|0\rangle_A \rightarrow |1\rangle_P|1\rangle_A \quad \Leftrightarrow \quad \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \rightarrow \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

$$|1\rangle_P|1\rangle_A \rightarrow |1\rangle_P|0\rangle_A \quad \Leftrightarrow \quad \begin{pmatrix}0\\0\\0\\1\end{pmatrix} \rightarrow \begin{pmatrix}0\\0\\1\\0\end{pmatrix}$$

What transforms vectors... matrices!

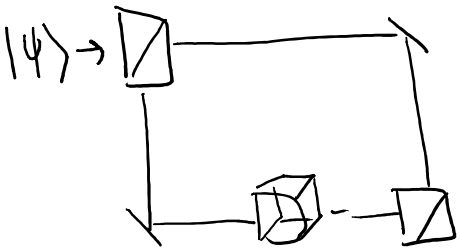
$$U_{BS} = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$$

* only certain types of matrices are allowed. Will discuss which later

New part of Quantum Mechanics: Quantum Operations "Gates"

S. KIMMEL

Effect of Bomb in Path



Bomb measures which arm photon is in

$|1\rangle_A \rightarrow$ explosion

$|0\rangle_A \rightarrow$ no explosion

If input state is

$$|\psi\rangle = (a|0\rangle + b|1\rangle)_p |0\rangle_A \xrightarrow{\text{Beamsplitter}} a|0\rangle_p|0\rangle_A + b|1\rangle_p|1\rangle_A$$

w/prob $|a|^2$ ↙ STATE COLLAPSE FROM BOMB ↘ w/prob $|b|^2$

$|0\rangle_p|0\rangle_A$

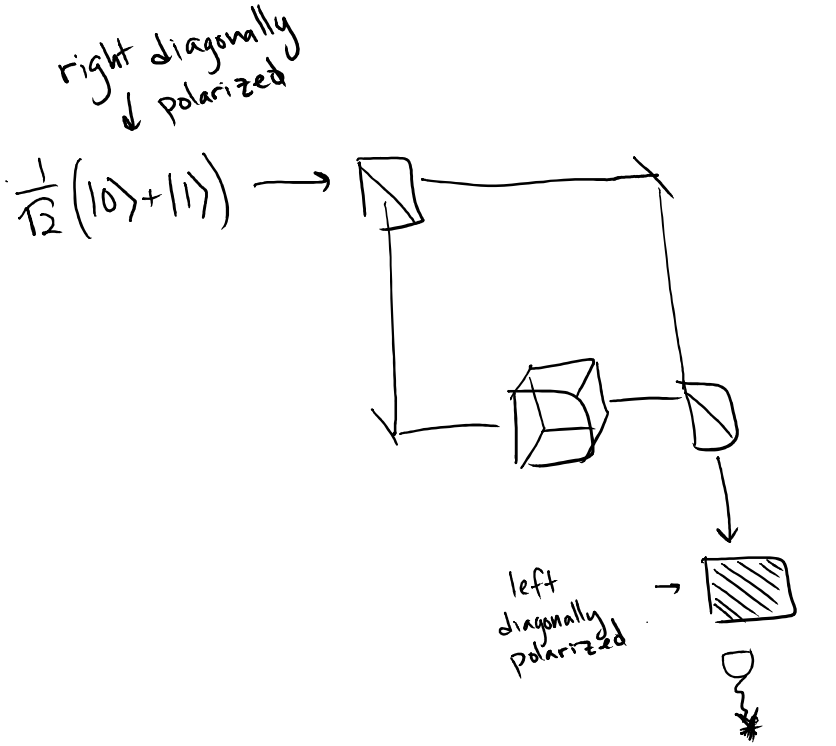
No explosion

$|1\rangle_p|1\rangle_A$

EXPLOSION

(This is a special case of "partial measurement." We will discuss general case for teleportation.)

S. KIMMEL

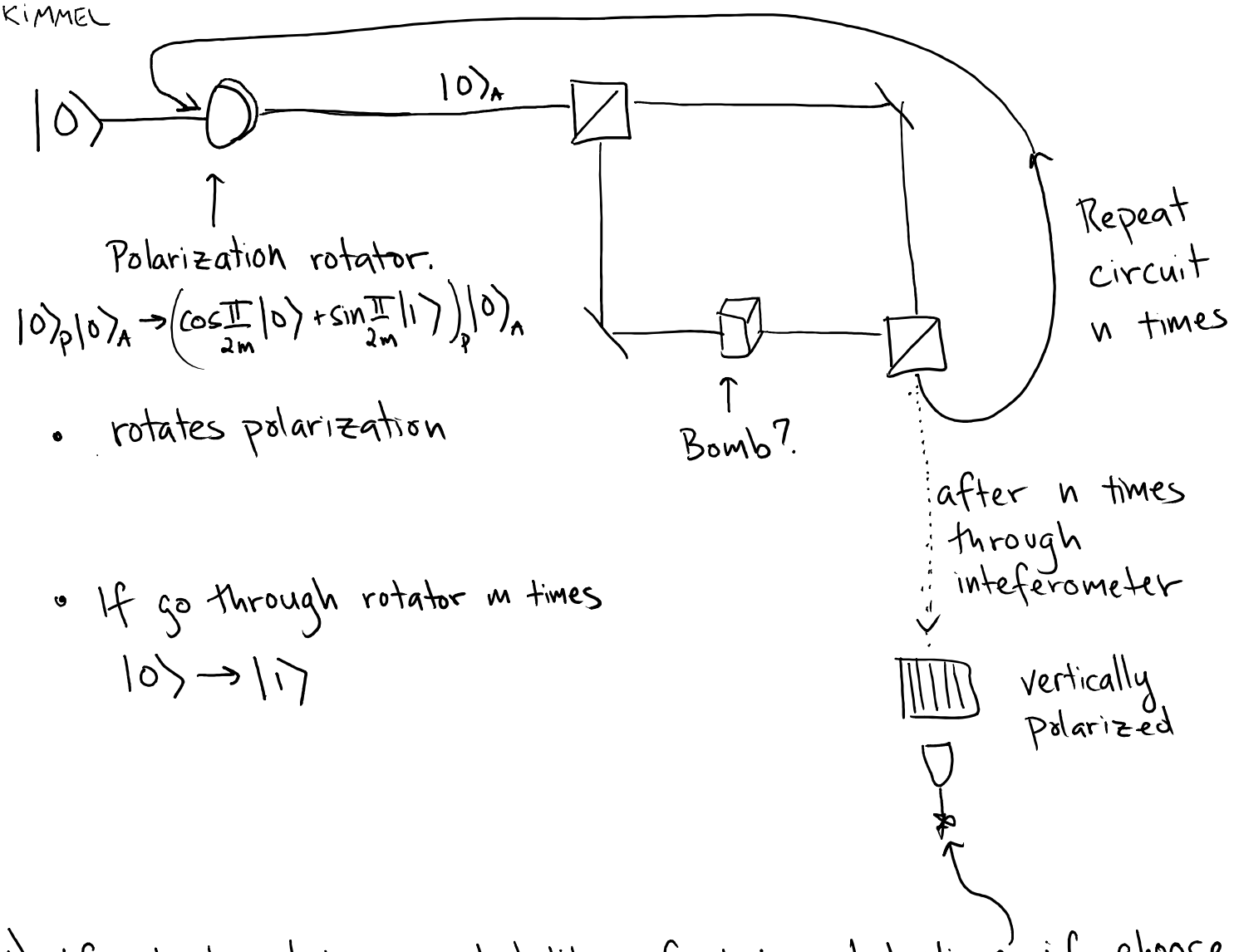


- Dud : 100% no detection light
- Bomb: 50% bomb explodes
  - 25% dectect photon
  - 25% no detection

There is a bomb, but you didn't blow it up! You can save it for a rainy day :)

Can detect an object without shining light on it!

S. KIMMEL

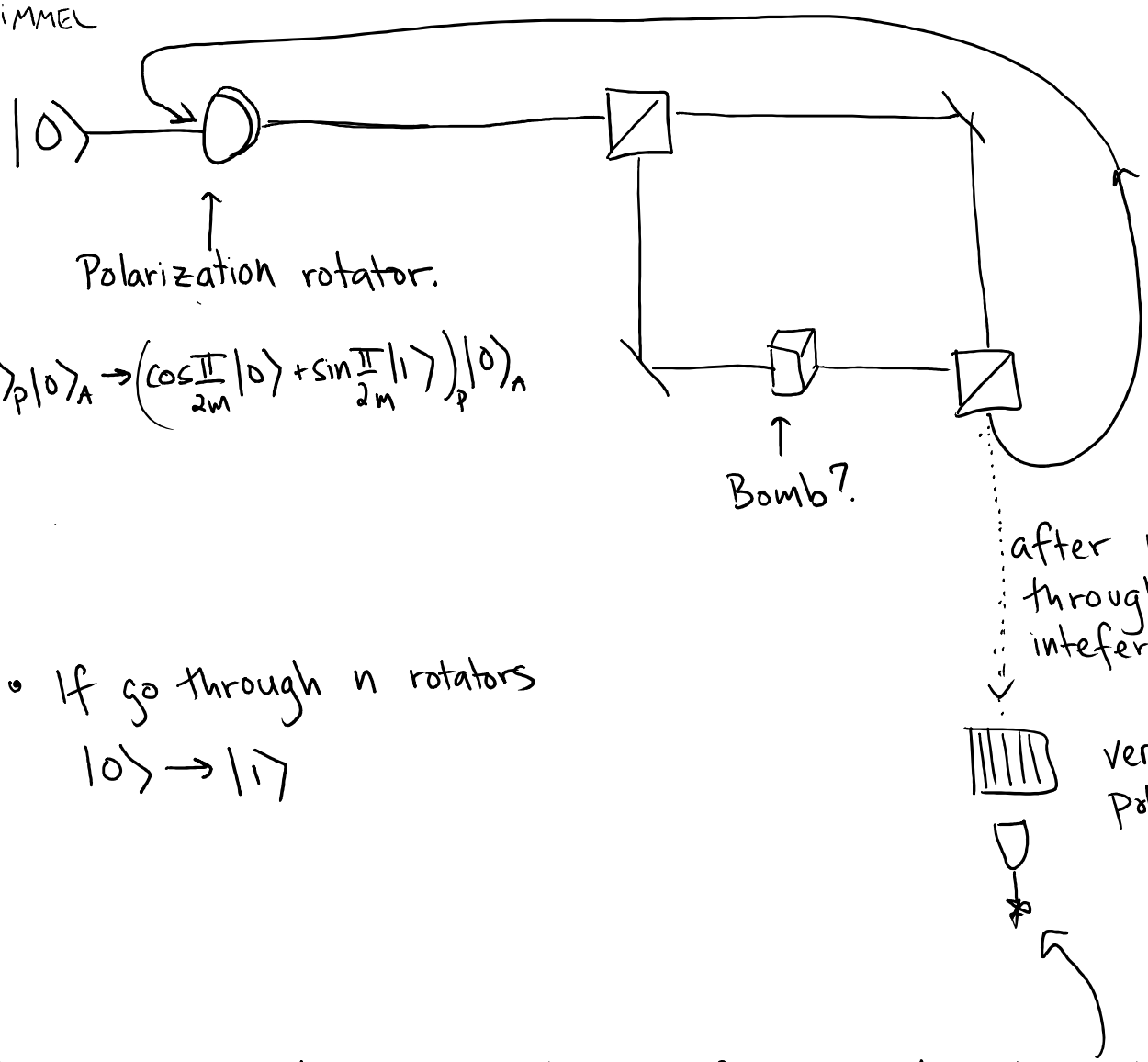


- rotates polarization

- If go through rotator m times
  $|0\rangle \rightarrow |1\rangle$

1) If dud, what is probability of photon detection if choose $m=n$?
2) If bomb, what is probability of no explosion?
3) If bomb but no explosion, what is probability of detection?

S. KIMMEL



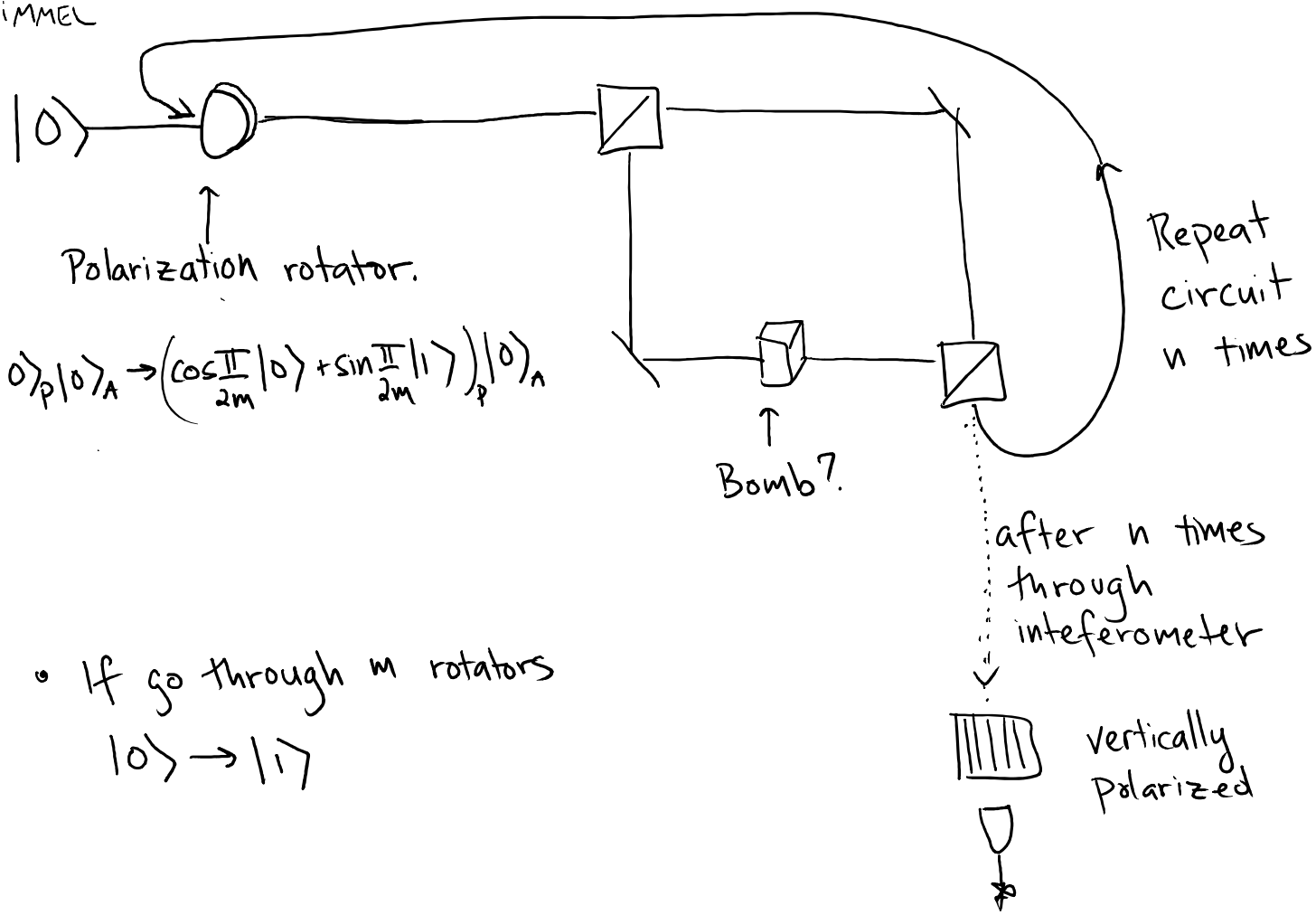
$|0\rangle_P|0\rangle_A \rightarrow \left(\cos\frac{\pi}{2m}|0\rangle + \sin\frac{\pi}{2m}|1\rangle\right)_P|0\rangle_A$

- If go through n rotators
  $|0\rangle \rightarrow |1\rangle$

1) If dud, what is probability of photon detection if choose m=n?

If dud, same polarization comes out of interferometer as went in. Then goes through rotator m times. $|0\rangle \rightarrow |1\rangle$. $|1\rangle$ is blocked by filter. No detection!

S. KIMMEL



- If go through m rotators
  $|0\rangle \rightarrow |1\rangle$

2) If bomb, what is probability of no explosion?

$|0\rangle_p|0\rangle_A \rightarrow \left(\cos\frac{\pi}{2n}|0\rangle + \sin\frac{\pi}{2n}|1\rangle\right)_p|0\rangle_A$ Rotator

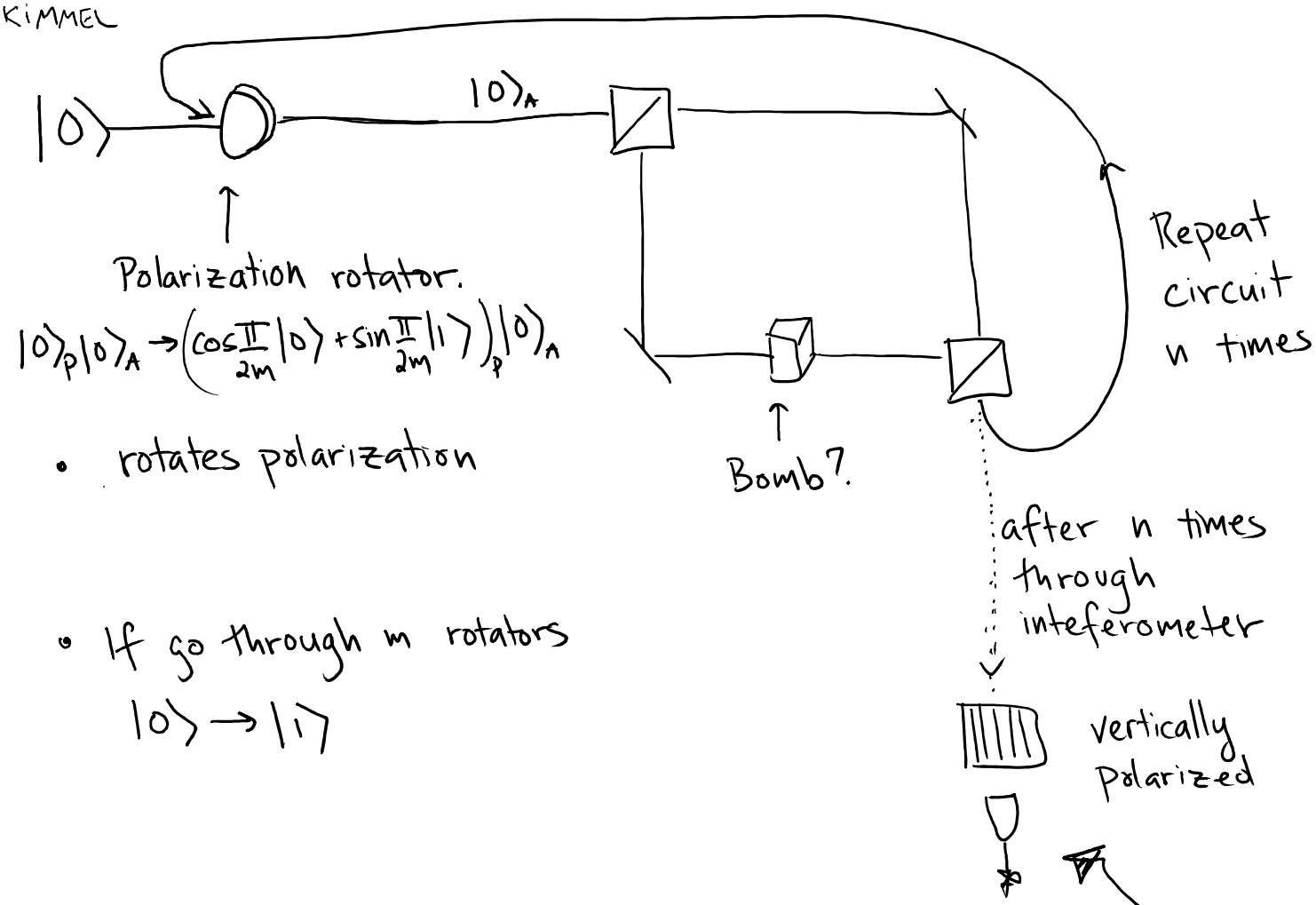
$\rightarrow \cos\frac{\pi}{2n}|0\rangle_p|0\rangle_A + \sin\frac{\pi}{2n}|1\rangle_p|1\rangle_A$ Beamsplitter

$\rightarrow |0\rangle_p|0\rangle_A$

Bomb, no explosion w/prob $\cos^2\left(\frac{\pi}{2m}\right)$

Prob never explodes: $\left(\cos^2\left(\frac{\pi}{2m}\right)\right)^n \approx 1$ if $n=m$ and $m,n \gg 1$.

S. KIMMEL



- rotates polarization

- If go through m rotators
  $|0\rangle \rightarrow |1\rangle$

3) If bomb but no explosion, what is probability of photon detection?

From part 2, we saw if bomb but no explosion, $|0\rangle_p$ always exits → will give detection

---

If choose $n = m \gg 1$,

Result: If No Bomb: No Detection light

If Bomb: Almost never explodes. If doesn't explode, detection light