S.KIMMEL

Runtime

Initialize each $i \in \Sigma$ as tree

While (> 1 tree to be merged)

- Find 2 trees with smallest probability
- Merge into new tree with new probability = sum of old probabilities

Q. What is the runtime?

A) $O(n)$ B) $O(n \log n)$ C) $O(n^2)$ D) $O(n^2 \log n)$

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Runtime

Initialize each $i \in \Sigma$ as tree ← $O(n)$

While (> 1 tree to be merged) ← $O(n)$ reps

- Find 2 trees with smallest probability ← Extract twice $O(\log n)$
- Merge into new tree with new probability → $O(1)$
  = Sum of old probabilities
- Reinsert into heap : $O(\log(n))$

Create 2 new pointers

Q. What is the runtime?

A) $O(n)$ B) $O(n \log n)$ C) $O(n^2)$ D) $O(n^2 \log n)$

↑ Keep finding min over in over in a changing data structure

Use min-heap

- Initialize n elements in $O(n)$
- Extract min elt in $O(\log n)$
- Insert a new elt in $O(\log n)$

Using different data structure, can acheive $O(n \log\log n)$

van Emde Boas tree

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Probability Questions

ex: n coin tosses, 1/4 prob of heads. How many heads on average?

1. Sample space

$\{H,T\}^n$ = all strings of length n consisting of heads tails

2. What is key random variable (look at question)

X = # of heads

3. Write as sum of indicator random variables

$X = \sum X_i$ $\quad X_i = \begin{cases} 1 & \text{if } i^{th} \text{ is head} \\ 0 & \text{if tails} \end{cases}$

(H T T H)

$X_1 = 1$
$X_2 = 0$
$X_3 = 0$
$X_4 = 1$

$\sum X_i = 2$ = # of heads

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3. $E[X] = E[\sum X_i] = \sum_i E[X_i]$

* For indicator random variables,

$E[X_i] = Pr(\text{Event associated with 1-value of } X_i)$

$\downarrow$

$Pr(\text{Get heads on flip } i)$

$= 1/4$

$E[X] = \sum_{i=1}^{n} 1/4 = \frac{n}{4}$

Each group prepare to explain $\frac{2}{|j-i|+1}$ ... random

(Each person must say something.)

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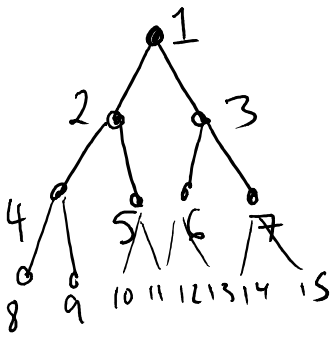
MWIS on a binary tree



Weights

10	5	1	9		
1	2	3	4	...	

1 Options for optimal soln:

(i) 1 is in MWIS

(ii) 1 is not in MWIS

2. Form of solution as recurrence relation

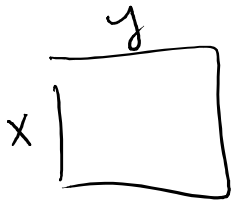
(i) $S(T_1) = S(T_4) + S(T_5) + S(T_6) + S(T_7) + 1$

(ii) $S(T_1) = S(T_2) + S(T_3)$

Can be a function of multiple variables:

- Knapsack: Capacity, items

- Aluminum sheets: Dimension A, Dim B



- DC/Washington: Week, current location

only 2 options