CS302 - Problem Set 11

1. **[6 points]** What is the worst case runtime of the following algorithm for depth-first search? Explain.

Algorithm 1: `DepthFirstSearch`(A, X, s, f)

Input : Adjacency list A for a graph $G = (V, E)$, an array X of length $|V|$ such that $X[v] = 1$ if v has been explored and 0 otherwise, a starting vertex s, a goal vertex f

Output: String "f found!" or "f not found" depending on whether f can be found from s.

```
if s == f then
    Return "f found!";
else
    X[s] = 1;
    d = A[s].length;
    for k = 1 to d do
        if X[A[s,k]] == 0 then
            DepthFirstSearch(A, X, A[s,k], f);
        end
    end
end
Return "f not found"
```

2. Given a path from s to t in a graph $G = (E, V)$, we define the bottleneck of the path to be the maximum weight of any edge on the path. We would like to create an algorithm that, given a starting vertex s, for every other vertex v in the graph finds the path with the smallest bottleneck from s to v, and returns the value of the bottleneck on that path.

   (a) **[9 points]** Describe (using pseudocode) a modification of Dijkstra's algorithm that solves this problem. (Hint - the algorithm stays the same, just the criterion changes.) I've started it for you:

   Algorithm 2: `BottleNeck`(A, s)

   Input : Graph $G(V, E)$ with positive edge weights $l(u, v)$, and a vertex $s \in V$.

   Output: Array B such that $B[v]$ is the path from s to v that has the minimum bottleneck of all paths from s to v, and an array A such that $A[v]$ is the value of the bottleneck of the path $B[v]$.

   (b) **[11 points]** Prove your algorithm finds the path with smallest bottleneck from s to every other vertex in G.

3. *Detecting Negative Cycles*

(a) [**6 points**] In the version of Bellman-Ford we looked at in class, we assume that there are no negative cycles in the graph. Describe using words how you would change the Bellman Ford algorithm either detect a negative cycle, or return the shortest path if there are no negative cycles. Also explain why your modification is correct.

(b) [**6 points**] What is the runtime of your modified algorithm. Explain.