

CS302 Midterm 1 Review

1. Create a loop invariant for the loop at Line 7 that will allow you to prove that if A_1 and A_2 are sorted arrays which together contains all elements of A , then the output of the algorithm is A , sorted. Write termination, base case, and maintenance conditions.

Algorithm 1: MergeSort(A)

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Input : Integer array  $A$  of length  $n$ 
Output: Sorted array
// Base Case
1 if  $n == 1$  then
2   | return  $A$ ;
3 end
// Divide and Conquer
4  $A_1 = \text{MergeSort}(A[1 : n/2])$ ;
5  $A_2 = \text{MergeSort}(A[n/2 + 1 : n])$ ;
// Combine
6  $p_1 = p_2 = 1$ ;
7 for  $i=1$  to  $n$  do
8   | if  $A_1[p_1] < A_2[p_2]$  then
9     |    $A[i] = A_1[p_1]$ ;
10    |    $p_1++$ ;
11   | else
12     |    $A[i] = A_2[p_2]$ ;
13     |    $p_2++$ ;
14   | end
15 end
16 return  $A$ 
```

2. Suppose you have n events, each with a start time s_i and end time f_i , for $i \in \{1, \dots, n\}$. Unfortunately, you only have one auditorium, and you can't schedule conflicting events (events where a start time of one is between the start time and end time of another.) We would like to maximize the number of events that are held. Show that a greedy algorithm that always chooses the next event that has the earliest possible starting time is optimal. You may assume that there is exactly one optimal solution (to make the exchange argument the single exchange type.)