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Learning Goals

- Describe MWIS & Scheduling problems
- Design a recurrence relation for MWIS
- Understand process of designing and testing a greedy algorithm.

Sample Syllabus Quiz Question:

Q: Which of the following problem set parts are graded for correctness?

A. Rough Draft.

B. Main PSet Submission

C. Self Grade

D. None of them ⟸ All graded on effort, although you will get the most out of self-grade if you try for accuracy.

Assorted Stuff

- Rough Draft (due today) — anything that shows some effort
- PSet Due Sunday @ 9pm (tutoring 7-9 Sunday, Wed)
- Syllabus Quiz Monday
- Scheduling Changes → will discuss next week

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<u>Divide + Conquer:</u>

Recursive algorithm

$\Downarrow$

Recurrence Relation

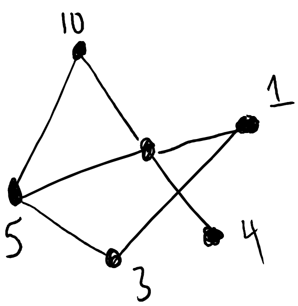
<u>Dynamic Programming</u>

Recurrence Relation

$\Downarrow$

Algorithm

<u>Max-Weight Independent Set Problem</u> (MWISP)



Input: Graph (V, E) and weight function $w: V \to \mathbb{Z}^+$

vertices edges

Output:
- $S \subseteq V$ s.t. if $(v_i, v_j) \in E$, v_i, v_j can't both be in S. } Independent set
- $w(S) = \sum_{v \in V} w(v)$ is maximized } max weight

This set is Max Weight Ind. Set (MWIS)

"objective function"

<u>Applications</u>

- Wifi transmitters / cell towers
  - Tower i has $n(i)$ packets to broadcast
  - If two towers are within 2 miles and broadcast at same time, causes interference

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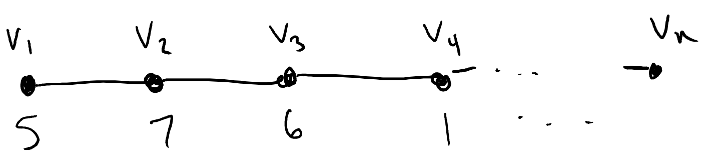
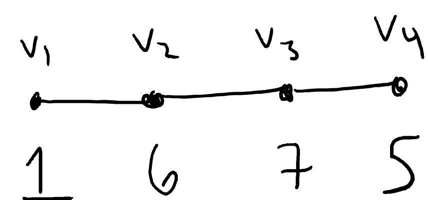
Q: How to use MWIS to determine which towers should transmit? What is V, E? What is $w: V \to \mathbb{R}$?

vertex set edge set

MWISP on Path Graph



Q: What is MWIS of



A) 11 B) 13 C) 19 D) None exists

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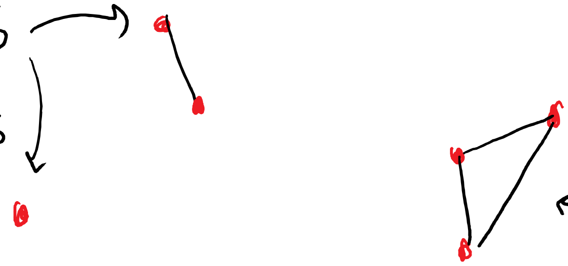
Q: How to use MWIS to determine which towers should transmit? What is V, E? What is $w : V \to \mathbb{R}$?

vertex set   edge set

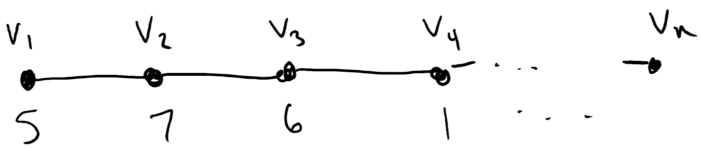
Vertices are towers



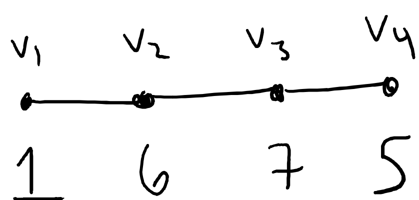
Put an edge b/t any two vertices that are 2 miles or less apart

$w(i) = n(i)$

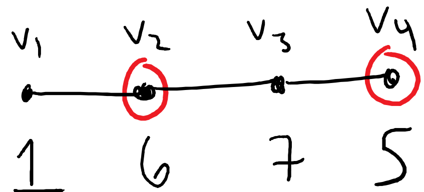
MWISP on Path Graph



Q: What is MWIS of



A) 11   B) 13   C) 19   D) None exists



$w(\{v_1, v_3\}) \to 8$

$w(\{v_2, v_4\}) \to 11$

$w(\{v_1, v_4\}) \to 6$

Best

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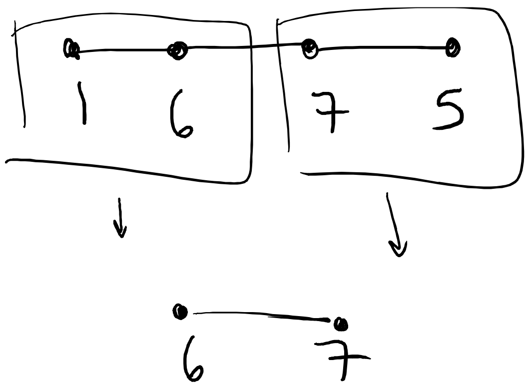
Divide & Conquer



can't combine!

* Can create a divide & conquer alg., but performance not optimal

Instead: Let's create a recurrence relation for the MWIS.

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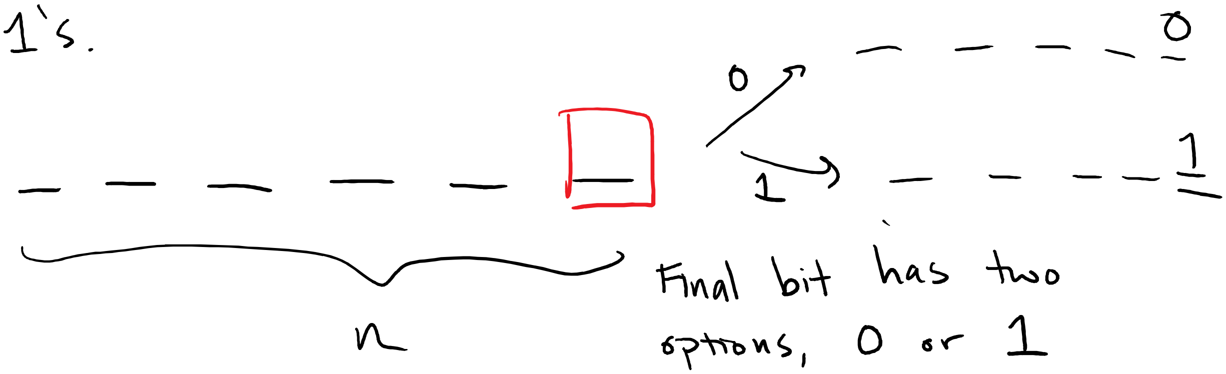
- Previous Example of Recurrence Relation
  Let $T(n)$ be # of n bit strings with even # of 1's.

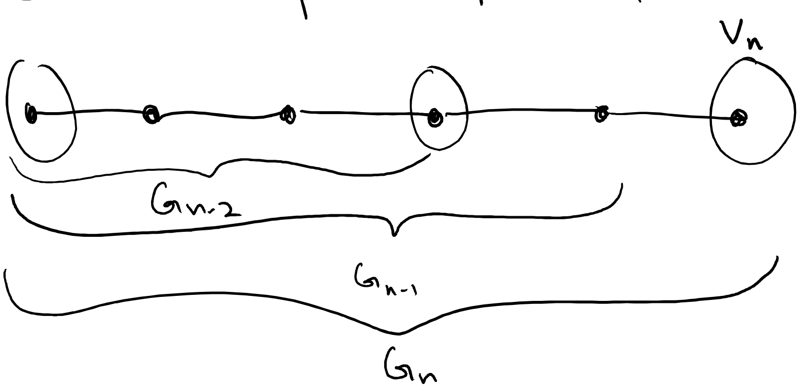


Final bit has two options, 0 or 1

$$T(n) = \underbrace{T(n-1)}_{\text{if } 0} + \underbrace{\cdots}_{\text{if } 1}$$

Now to MWIS Problem:

1. Consider options for optimal solution S

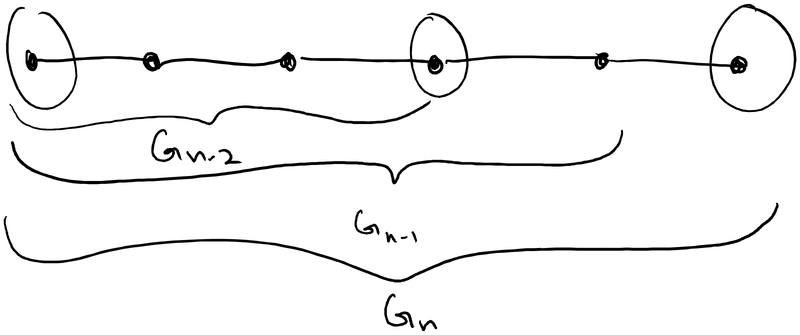


← Last vertex has two options

i) $v_n \notin S$

ii) $v_n \in S$

2. Relate S to solution of smaller problem for each option



← last vertex has two options

i) $v_n \notin S$

ii) $v_n \in S$

i) If $v_n \notin S$. $\square$ is MWIS on the graph $\square$.

Why? (Try proof by contradiction.)

ii) $v_n \in S$, $\square$ is MWIS on the graph $\square$

Why? (Try proof by contradiction.)

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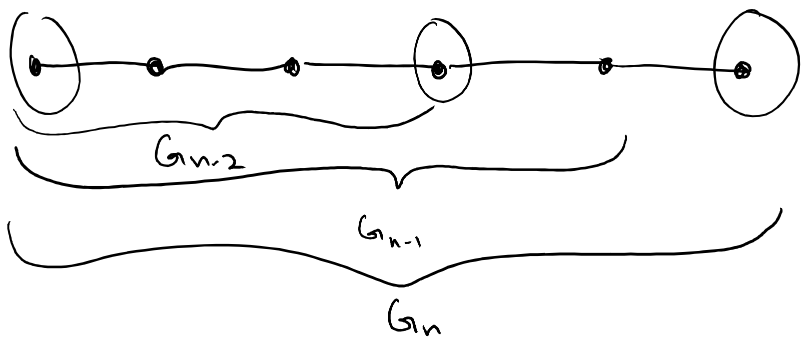
2. Relate S (← MWIS on G_n) to solution of smaller problem for each option



← last vertex has two options

i) $v_n \notin S$

ii) $v_n \in S$

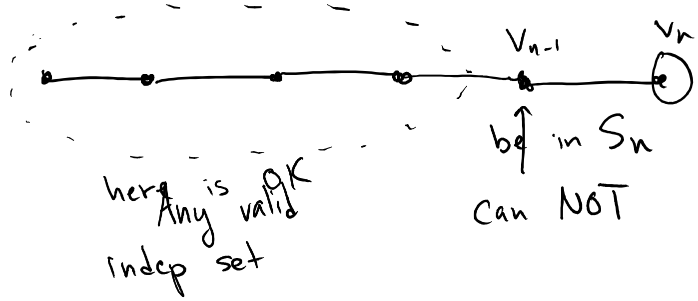
i) If $v_n \notin S$. S is MWIS on G_{n-1}

Pf: • S is an ind. set of G_{n-1} so just need to show has max weight

• Suppose for contradiction S' is an ind. set of G_{n-1} with larger weight than S. Then S' is also a larger weight ind. set on G_n than S, so S is not MWIS of G_n, a contradiction

ii) $v_n \in S$, $S - v_n$ is MWIS on G_{n-2}

Pf • $S - v_n$ is a valid ind. set for G_{n-2}, so need to show has max weight



• Suppose for contradiction S' is an ind. set of G_{n-2} with larger weight than $S - v_n$. Then $S' \cup v_n$ will have larger weight than S on G_n, a contradiction