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Goals

- Prove correctness of greedy scheduling
- Prove correctness of loops using loop invariants

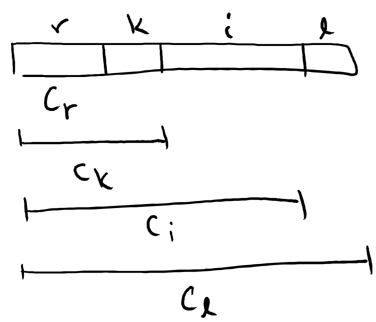
Recall

Scheduling: n jobs

t_i = time to run job i

w_i = weight (importance) of job i

C_i = time to complete job i



Want to minimize $A = \sum_i w_i C_i$

We've tried several greedy algorithms, we think

$f(w,t) = \frac{w}{t}$ is optimal (order by largest f)

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Thm: Greedy algorithm with $f = w_i/t_i$ is optimal for objective function $\sum_i w_i C_i$.

Pf: EXCHANGE argument (Proof by contradiction)

Assume w_i/t_i are distinct $\forall\ i \in \{1, 2, \dots, n\}$

WLOG, relabel so $w_1/t_1 > w_2/t_2 > \cdots > w_n/t_n$

Let σ be ordering using greedy, so $\sigma = (1, 2, 3, \dots, n)$

For contradiction assume that the optimal ordering σ^* is not the greedy ordering.

Note: $\exists\ k, j$ s.t. $w_j/t_j > w_k/t_k$, but j is immediately after k in σ^* ordering.

(Otherwise, $\sigma^* = \sigma$!)

Let's create a new ordering $\sigma^{*\prime}$ that is same as σ^*, but with k, j positions switched

Q: If σ^* has objective value A_{σ^*}, and $\sigma^{*\prime}$ has objective value $A_{\sigma^{*\prime}}$, what is $A_{\sigma^*} - A_{\sigma^{*\prime}}$?

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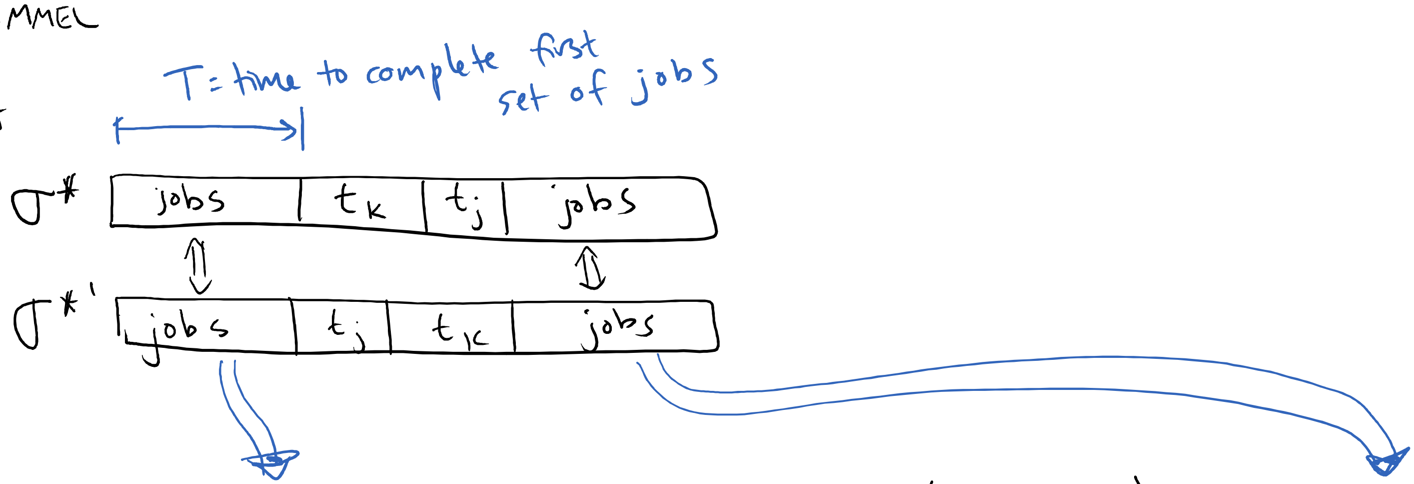
A



$$A_{\sigma^*} = \sum w_i C_i + w_k(T + t_k) + w_j(T + t_k + t_j) + \sum w_r C_r$$

$$A_{\sigma^{*'}} = \sum w_i C_i + w_j(T + t_j) + w_k(T + t_j + t_k) + \sum w_r C_r$$

$$\boxed{A_{\sigma^*} - A_{\sigma^{*'}} = w_j t_k - w_k t_j}$$

$$\frac{w_j}{t_j} > \frac{w_k}{t_k} \implies w_j t_k > w_k t_j$$

$A_{\sigma^{*'}} < A_{\sigma^*} \implies$ Contradiction!

(Because σ^* is optimal it should have smallest A.)

Thus our assumption that σ was not optimal was incorrect and σ is optimal schedule

Q: What is the runtime of the greedy scheduling alg.?

A) $O(1)$ B) $O(n)$ C) $O(n \log n)$ D) $O(n^2)$

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Q: What is the runtime of the greedy scheduling alg.?

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need to sort by f-value

Structure of Exchange Proof

1. Assume for contradiction there is a optimal strategy that is not your greedy one.
2. The optimal strategy will differ from greedy somehow, so can create a modifed strategy that usually exchanges elements of optimal to make it more like greedy
3. Show modified strategy is better than optimal, a contradiction.
4. Therefore, greedy is optimal.