Section A

| | | |
|---|---|---|
| Agastya | Loic | Leili |
| Sam K | Hirona | Sam L |
| Cliff | Danielle | Halcyon |
| | | |
| Dubner | Sammy | Samantha |
| Kelly | Benjamin | Lily |
| Alexander | Diego | Lucas |
| | | |
| Kaylee | Cheko | |
| Yaqi | Quinlan | |
| Sam C | Culton | |

Section B

| | | |
|---|---|---|
| Diana | Matt | Gray |
| Maddy | Christian | Alexandra |
| Christi | Alexander | Caroline |
| | | |
| Stephan | Rachel | Musab |
| Griffin | Sophie S | Jacob |
| Max | Myles | Angad |
| | | |
| Arielle | Aska | |
| Henry | Matthew | |
| Hannah | Sophie E. | |

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# Goals

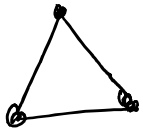
- Describe graph properties (bipartite, directed, weighted)
- Use adjacency matrices & adjacency lists to describe graphs.
- Write pseudocode to analyze graph properties.

## • Bipartite

$G=(V,E)$ is bipartite $\equiv$

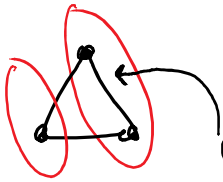
$$\exists\, S,T \subseteq V : S \cup T = V \;/\!\!\backslash\; S \cap T = \emptyset \wedge \left( \forall\, a,b \in V, \left( (a,b \in S) \vee (a,b \in T) \right) \rightarrow \{a,b\} \notin E \right)$$

Q: Is this graph bipartite?



A) Yes

B) No ⟸



edge between vertices in same set

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- **Directed Graph**

$G = (V, E)$

$V$ = set of vertices

$E$ = set of edges

An edge $e = (a, b)$

ordered pair of vertices


✡ For directed graphs $E \subseteq V \times V$, so $E$ is a relation

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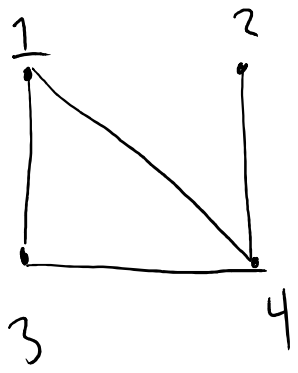
# Ways to Represent Graphs in Computer

## Adjacency Matrix

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 1 |
| 2 | 0 | 0 | 0 | 1 |
| 3 | 1 | 0 | 0 | 1 |
| 4 | 1 | 1 | 1 | 0 |




Store as array A in memory. E.g. $A[3,4] = 1$

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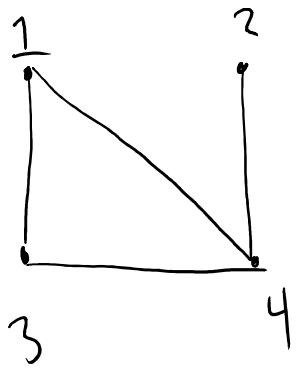
# Adjacency List

| Vertex | Adjacent Vertices |
|---|---|
| 1 | 3, 4 |
| 2 | 4 |
| 3 | 1, 4 |
| 4 | 1, 2, 3 |




Store as an array of lists

ex: $A[4] = (1, 2, 3)$

$A[3, 2] = 4$

$A.length = 4$ or $length(A) = 4$

$A[3].length = 2$ $length(A[3]) = 2$

Pseudocode! As long as clear, can do anything

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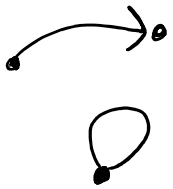
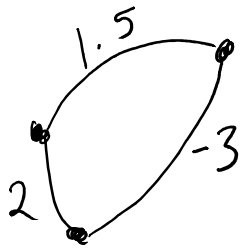
How would you represent a

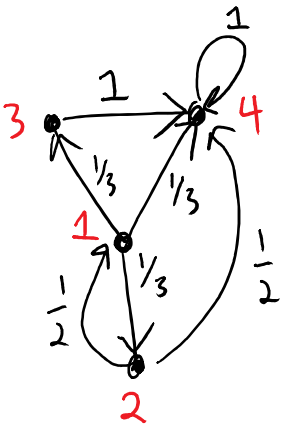
- directed graph?
- graph with self-loops?
- graph with multi edges?
- graph with weighted edges?






Using Adjacency Matrix / Adjacency List?

Give representations of this graph using both approaches:




| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0 | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ |
| 2 | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ |
| 3 | 0 | 0 | 0 | 1 |
| 4 | 0 | 0 | 0 | 1 |

| V | List |
|---|---|
| 1 | $(2, \frac{1}{3}), (3, \frac{1}{3}), (4, \frac{1}{3})$ |
| 2 | $(1, \frac{1}{2}), (4, \frac{1}{2})$ |
| 3 | $(4, 1)$ |
| 4 | $(4, 1)$ |