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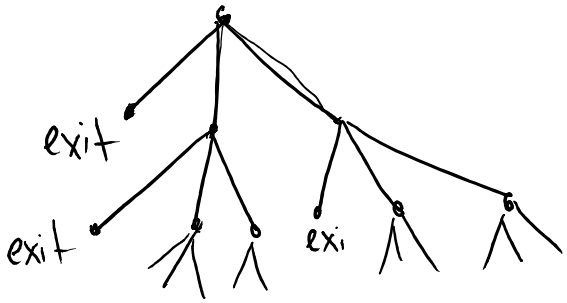
# Using Trees for Probability:
## When Trees Get Big... What To Do

ex: You are spelunking and come across a strange series of rooms. In the first room there are 3 passages: one of which exits the cave and the other two lead to a new rooms, each of which has 3 passages: one exits the cave and the other two lead to a new rooms

Consider possible outcomes if each passage is chosen at random:




Infinite Size tree!

What is probability that you visit at least 5 new rooms or exactly 1 room?

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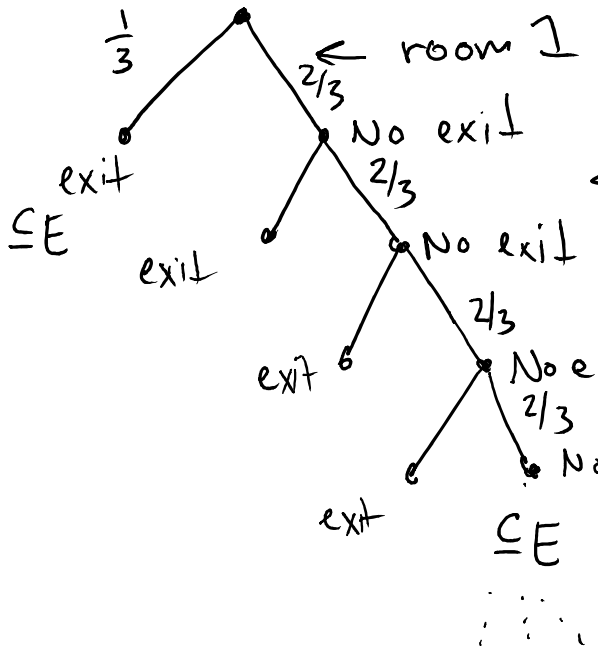
- If sample space is too fine grained for what you need, combine options, as long as each outcome is part of event or not

$E$ = 25 rooms or exactly 1




$\subseteq E$ $\subseteq E$ ...

every further outcome on this subtree is part of event. Can stop creating tree

Once all future possibility are part of an event, can stop creating more branches

$$P(E) = \frac{1}{3} + \frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3} = \frac{1}{3} + \left(\frac{2}{3}\right)^4$$

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Let $X$ = # of rooms visited before exit.

Write expression for $\mathbb{E}[X]$ (don't use indicator random variables). Sample space = {exit after 1 room, exit after 2 rooms, exit after 3 rooms... }

$$\mathbb{E}[X] = \sum_{i \in S} \Pr(i) X(i)$$

$$= \sum_{i=1}^{\infty} \frac{1}{3} \left(\frac{2}{3}\right)^{i-1} i$$