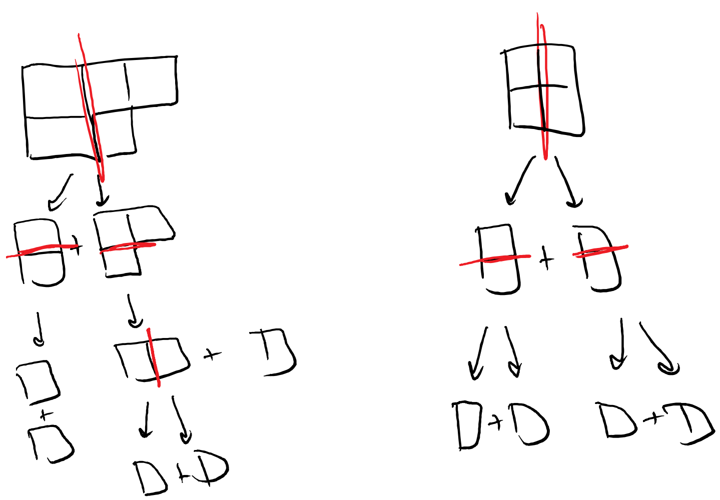
S.KIMMEL

Goals

- Write a strong inductive Proof
- Describe when multiple base cases necessary

Strong Induction

Q: Suppose you have a bar of chocolate containing n small joined squares. How many times do you have to break the chocolate along a row or column before you have n separate squares?



A) $n-1$

B) n

C) Depends on original shape

D) I want chocolate

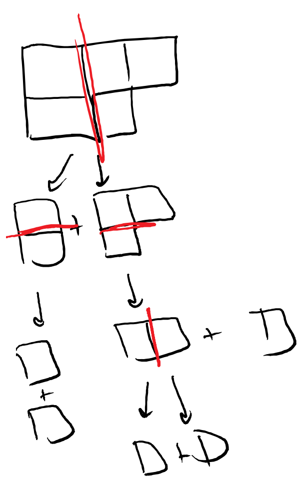
S.KIMMEL

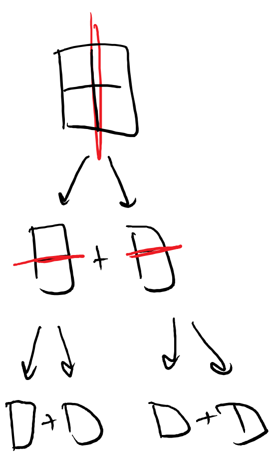
Goals

- Write a strong inductive proof
- Describe when multiple base cases necessary

Strong Induction

Q: Suppose you have a bar of chocolate containing n small joined squares. How many times do you have to break the chocolate along a row or column before you have n separate squares?





A) n-1 ← Both examples require n-1 Proof?

B) n

C) Depends on original shape

D) I want chocolate

Induction seems good, because after breaking, end up with smaller chocolate bars. If we knew how many breaks needed for smaller bars, we could use to solve bigger problem

BUT

Smaller bar might not have n-1 pieces

S.KIMMEL

Strong Induction Proof Structure

Set-up

Let P(n) be the predicate ____________________ We will prove P(n) is true for all $n \geq$ b.c.

(b.c. = base case)

Base-case

Base-case: We prove P(b.c.) is true. ... (Sometimes multiple b.c.) P(0), P(1))

Inductive case

Inductive case: Let $k \geq$ b.c. Assume for strong induction that P(j) is true for all j such that b.c. $\leq j \leq k$.

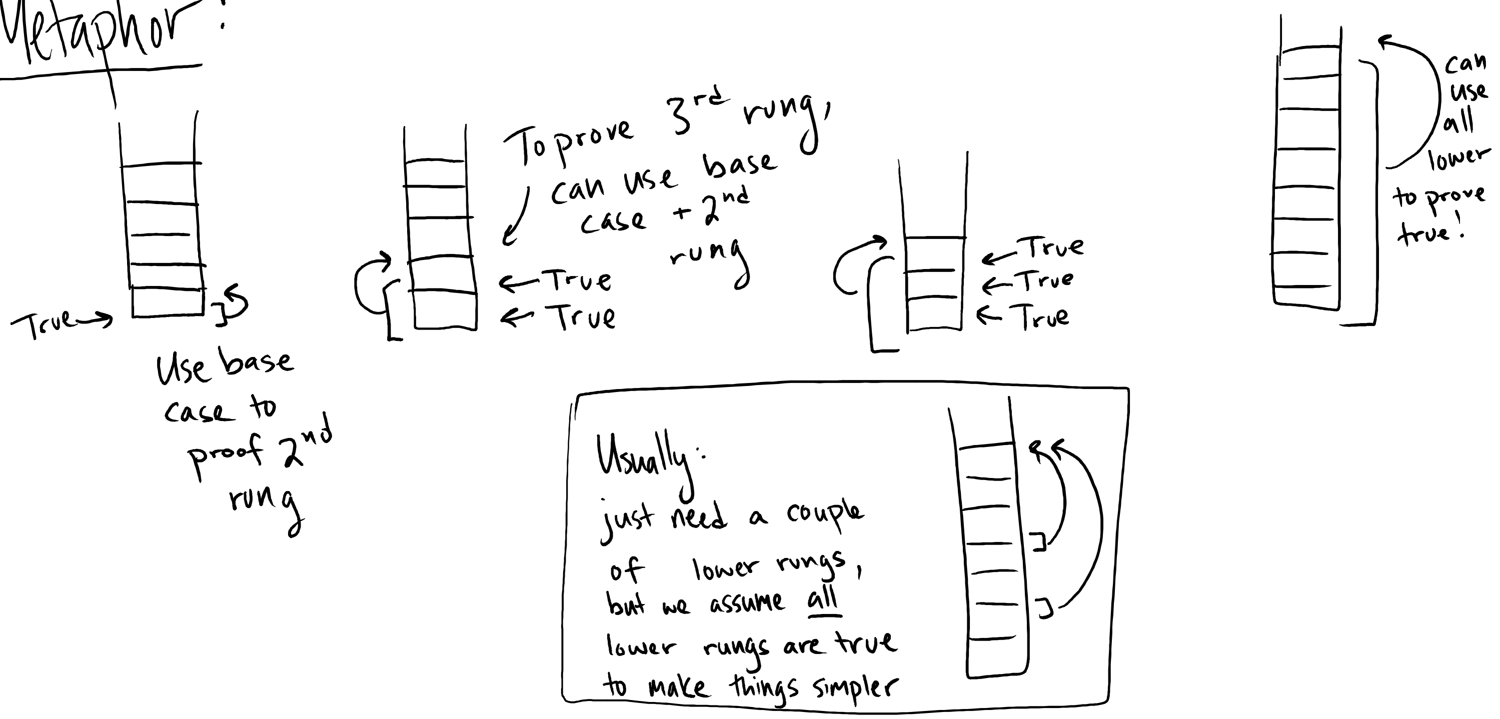
Explain

Therefore P(k+1) is true

Conclusion

By strong induction, we conclude P(n) is true for all $n \geq$ b.c.

Metaphor:



S. KIMMEL

Q: Prove it takes $n-1$ breaks to reduce an n-square chocolate bar to n individual squares.

A: Let $P(n)$ be the predicate " ." We will prove via strong induction that $P(n)$ is true for $n \in \mathbb{N}$, $n \geq 1$.

Base case: When you have a 1-square chocolate bar, it requires 0 breaks to create 1 individual squares, so $P(1)$ is true.

Inductive Case:
Let $k \geq 1$. We assume for strong induction that $P(j)$ is true for $1 \leq j \leq k$. We will prove $P(k+1)$ is true. Since $k+1 > 1$, we can break the chocolate into two pieces, one with a squares, and one with b squares, where $a+b = k+1$, and $1 \leq a \leq k$, and $1 \leq b \leq k$. Using our inductive assumption, it requires $(a-1)$ breaks to separate the first piece and $(b-1)$ breaks to separate the second. Adding up all the breaks, we have

$$(a-1) + (b-1) + 1 = a+b-1 = k$$

total breaks. Thus $P(k+1)$ is true.

Therefore, by strong induction, $P(n)$ is true.

S.KIMMEL

$F(n)$

1. If $n \leq 1$: return n
2. return $5 \cdot F(n-1) - 6 \cdot F(n-2)$

Q: Prove this algorithm returns $3^n - 2^n$ for all $n \geq 0$.

[only up to inductive step setup]

Let $P(n)$ be the predicate $F(n)$ returns $3^n - 2^n$. We will prove $P(n)$ is true for all $n \geq 0$, using strong induction

Base cases : We will show $P(0)$ and $P(1)$. When the input is 0, we return 0. Since $3^0 - 2^0 = 1 - 1 = 0$, this is correct. When the input is 1, we return 1. Since $3^1 - 2^1 = 3 - 2 = 1$, this is correct.

Inductive step: Let $\boxed{k \geq 1}$. Assume $P(j)$ is true for all j such that $\boxed{0 \leq j \leq k}$. We will prove $P(k+1)$

We want $k+1$ to be larger than base case, so choose k to be larger than or equal to largest base case

Want to assume all base cases are true, so j starts at smallest base case.

We need to prove $P(0)$ and $P(1)$. Otherwise when try to prove $P(2)$, look at $f(2-1) = f(1)$ and $f(2-2) = f(0)$, need to assume these output correctly