S.KIMMEL

Goals

- Prove (not) equivalence relation
- Identify equivalence classes
- Analyze runtime of FOR-loops

Spring Symposium
Bonus!

Reflections

- This pset <u>is</u> challenging
  - identify proof strategy
    - ✱ If stuck, try different approach
    - ✱ Will develop intuition with practice
  - combining functions/counting/graphs
    - ✱ Make sure basics are solid

Equivalence Relations

Why do we care? They allow us to define <u>equivalence classes</u>.

R is an equivalence relation on S (i.e. $R \subseteq S \times S$) $\Longleftrightarrow$ S can be divided up into equivalence classes according to R.

$(a,b) \in R$ $\Longleftrightarrow$ a and b are in the same equivalence class

S. KIMMEL

Q: Let $S = \{0,1,2,3,4\}$. Let $R \subseteq S \times S$ be the equivalence relation:

$$R = \{(0,0), (0,1), (1,0), (1,1), (2,2), (3,3), (3,4), (4,3), (4,4)\}$$

What are equivalence classes?

A) $\{0,1,3,4\}$

B) $\{0,1\}, \{3,4\}$

C) $\{0,1\}, \{2\}, \{3,4\}$

D) $\{(0,0),(1,1)\}, \{(2,2)\}, \{(3,3),(4,4)\}$

S. KIMMEL

Q: Let $S = \{0,1,2,3,4\}$. Let $R \subseteq S \times S$ be the equivalence relation:

$$R = \{(0,0), (0,1), (1,0), (1,1), (2,2), (3,3), (3,4), (4,3), (4,4)\}$$

What are equivalence classes?

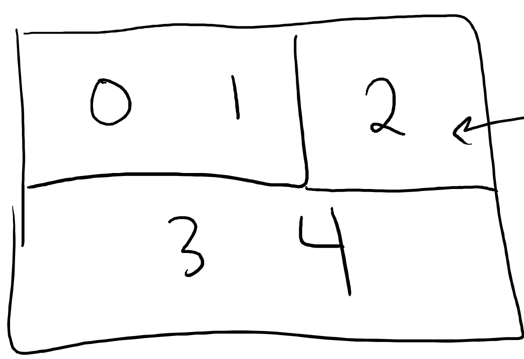
A) $\{0,1,3,4\}$

B) $\{0,1\}, \{3,4\}$

C) $\{0,1\}, \{2\}, \{3,4\}$

D) $\{(0,0),(1,1)\}, \{(2,2)\}, \{(3,3),(4,4)\}$

Divide



each is an equivalence class

Equivalence classes are subsets of S.

S.KIMMEL

To <u>prove</u> R is equivalence relation:

- 3 mini proofs (usually 1 line each) or explanations
  - Symmetric, Reflexive, Transitive

To prove R is not an equivalence relation

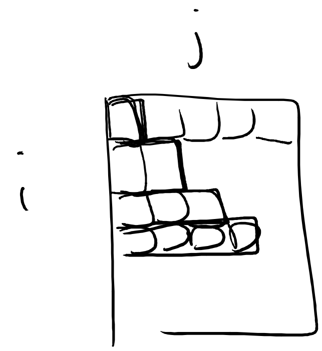
- Show <u>1</u> property doesn't hold
- Properties have form $\forall a \in S \ldots$

$\Rightarrow$ Need counterexample to show <u>not</u> $\forall a \in S \ldots$

Input: Adjacency Matrix A for $G=(V,E)$, G unweighted

Output: ??

```
1. S=0
2. for i=1 to |V|:
3.     for j=1 to i:
4.         S += A[i,j]
5. return S
```



This time doesn't over count! Returns $|E|$

How many operations?

- Use $\sum$ for loops
- Use 1 for $O(1)$ operations

$$\#\text{operations} = 1 + \sum_{i=1}^{|V|}\left[\text{work done inside } i^{th} \text{ loop iteration}\right]$$

line 1 & 5 line 2

$$= 1 + \sum_{i=1}^{|V|}\left[\sum_{j=1}^{i} 1\right]$$

line 3 line 4

Write your expression from outer loops to inner loop

S.KIMMEL

Evaluate from the inside out:

$$\# \text{ operations} = \underline{1} + \sum_{i=1}^{|V|} \left[\sum_{j=1}^{i} \underline{1} \right]$$

$$= \underline{1} + \sum_{i=1}^{|V|} i$$

$$= \underline{1} + \left[1 + 2 + 3 + \dots + |V| \right]$$

$$= 1 + \frac{(|V|+1)|V|}{2}$$

You proved when we did induction! "Arithmetic Series"

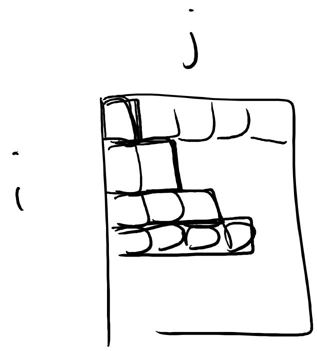
$$= O(|V|^2)$$

"Detailed Calculation"

S. KIMMEL

Input: Adjacency Matrix A for $G=(V,E)$, G unweighted
Output: ??

```
1. S = 0
2. for i = 1 to |V|:
3.     for j = 1 to i:
4.         S += A[i,j]
5. return S
```



This time doesn't over count! Returns $|E|$

How many operations?

Worst-case Calculation:

line 2: Repeats $|V|$ times:
- ↳ line 3: Worst case repeats $|V|$ times because $i \le |V|$
  - ↳ Does constant work

Nested loops: $O(|V| \times |V|)$
+ Constant

$$= O(|V|^2)$$

Big-O is <u>upper</u> bound!