S. KIMMEL

Q: What is a recurrence relation for the runtime of this algorithm?

```
Thing(n)
  if n=1 or n=2: return n
  for i = 1 to n^2
      Print: "meh"
  return Thing(n-1) + Thing(n-2)
```

A) $T(1) = O(1)$; $T(n) = T(n-1) * T(n-2) + O(n^2)$

B) $T(1) = O(n)$; $T(n) = T(n-1) * T(n-2) + O(n^2)$

C) $T(1) = O(1)$; $T(n) = T(n-1) + T(n-2) + O(n^2)$

D) $T(1) = O(1)$, $T(2) = O(1)$; $T(n) = T(n-1) + T(n-2) + O(n^2)$

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Q: What is a recurrence relation for the runtime of this algorithm?

Thing(n)

if n=1 or n=2: return n $\leftarrow O(1)$

for i = 1 to n^2

  Print: "meh" $] \leftarrow O(n^2)$

multiplication, return

return Thing(n-1) * Thing(n-2) $\leftarrow T(n-1) + T(n-2) + O(1)$

A) $T(1) = O(1)$; $T(n) = T(n-1) * T(n-2) + O(n^2)$

B) $T(1) = O(n)$; $T(n) = T(n-1) * T(n-2) + O(n^2)$

C) $T(1) = O(1)$; $T(n) = T(n-1) + T(n-2) + O(n^2)$

$\boxed{\text{D)}}$ $\underbrace{T(1) = O(1); T(2) = O(1);}$ $T(n) = T(n-1) + T(n-2) + O(n^2)$

need 2 base cases because otherwise fall off the bottom of ladder

$T(1) = O(1)$

$T(2) = T(1) + T(0) + O(2^2)$

?

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$K(n)$ = set of n-digit #'s with an even # of 0 s.

$$K(n) = \begin{bmatrix}\text{even \# of 0's} \\ \text{end with } 0\end{bmatrix} + \begin{bmatrix}\text{even \# of 0's} \\ \text{end with not } 0\end{bmatrix}$$

$$\underbrace{\_\_\_\_\_\_\_\_\_\_\_\_}_{n-1}\ \underline{0} \qquad \underbrace{\_\_\_\_\_\_\_\_\_\_\_\_}_{n-1}\ \underline{1 \ldots 9}$$

← 9 options

Need this part to have odd # of 0's.

Need this part to have even # of 0's

$\Rightarrow K(n-1) \rightarrow 9K(n-1)$

$$\begin{bmatrix}\text{\# n-1 digit with} \\ \text{odd \# of 0's}\end{bmatrix} + \begin{bmatrix}\text{\# n-1 digit} \\ \text{with even \#} \\ \text{of 0's}\end{bmatrix} = \text{All} = 10^{n-1}$$

$K(n-1)$

10 options for each digit

$$= 10^{n-1} - K(n-1)$$

$$K(n-1) = 10^{n-1} - K(n-1) + 9K(n-1) = 10^{n-1} + 8K(n-1)$$

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Let $T(n,k)$ = # of strings in $\{0,1,2\}^n$ whose digits sum to k:

Options

A: $\_\ \_\ \_\ \_\ \_\ 0 \Leftarrow T(n-1,k)$

B: $\_\ \_\ \_\ \_\ \_\ 1 \Leftarrow T(n-1,k-1)$ ← rest needs to sum to $k-1$

C: $\underbrace{\_\ \_\ \_\ \_\ \_}_{n-1}\ 2 \Leftarrow T(n-1,k-2)$ ← rest needs to sum to $k-2$

$$T(n,k) = T(n-1,k) + T(n-1,k-1) + T(n-2,k-2)$$

Base cases

$T(n,0) = 1$ (all 0's string)

$T(n,\text{negative}) = 0$ (never will sum to negative #)