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Feedback
Group switch

Goals: translate between English + Math

Quantifiers : turn predicates into statements

Universal Quantifier : $\forall$ means "for all", "every"

ex: $\forall x \in \mathbb{N}, x > 0$ $\approx$ "for every natural number x, $x > 0$."

- $\forall x \in \mathbb{N}$: quantifier
- $x > 0$: Predicate
- $\forall x \in \mathbb{N}, x > 0$: statement (true)
- use comma to separate

Existential Quantifier: $\exists$ means "there exists" "there is"

ex: $\exists x \in \mathbb{N} : x < 0$ $\approx$ "there exists a natural number x such that $x < 0$"

- $\exists x \in \mathbb{N}$: quantifier
- $x < 0$: predicate
- $\exists x \in \mathbb{N} : x < 0$: statement (false)
- use colon "such that" to separate

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Q: Let $M(x,y)$ be the predicate that x is y's parent. Let S be the set of all people (who have ever lived). True or false:

$$\forall x \in S, \exists\, y \in S : M(x,y)$$

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Q: Let $M(x,y)$ be the predicate that x is y's parent. Let S be the set of all people. True or false:

$$\forall x \in S, \exists\, y \in S : M(x,y)$$

False: It says for every person x, there is some person y such that x is y's parent. In other words, it is saying every person is a parent, which is not true

How would you modify it to be true?

$$\forall x \in S, \exists\, y \in S : M(y,x)$$

This is saying every person has a parent.

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* Quantifier must have domain

~~$\forall x, x > 0$~~

$\forall x \in \mathbb{Z}, x > 0$

domain = a set

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Turning English Predicates Into Math:

ex:

$P(n) \equiv$ n cents postage can be formed from 5¢ and 8¢ stamps

$$P(n) \equiv \exists\, x, y \in \mathbb{Z} : x, y \geq 0 \wedge 5x + 8y = n$$

(common construction $\exists\, m : m \wedge m$)

Q: $m \mid n$ means m divides n. What is $2 \mid 6$?

A) True B) False C) 3 D) 1/3

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↑

2 divides 6

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ex:

$E(n)$ = n is even

$K(n)$ = all even integers are divisible by 2.

Want:

$K(n) \equiv \forall x \in$ (set of even #'s) ← don't have this

Instead:

$K(n) \equiv \underbrace{\forall x \in \mathbb{Z},\ E(n) \rightarrow 2 \mid n}$

does same thing

(common construction: $\forall \sim,\ \sim \rightarrow \sim$)

Why? When writing proofs, usually easier to manipulate math. Useful to be able to convert from English to math

Important

- For input to Predicate NO QUANTIFIER
- For any other variable NEED QUANTIFIER

↱ $S \equiv [\quad]$ ← all variables quantified

statement

x is quantified if write $\forall x \in \_$, or $\exists x \in \_$: