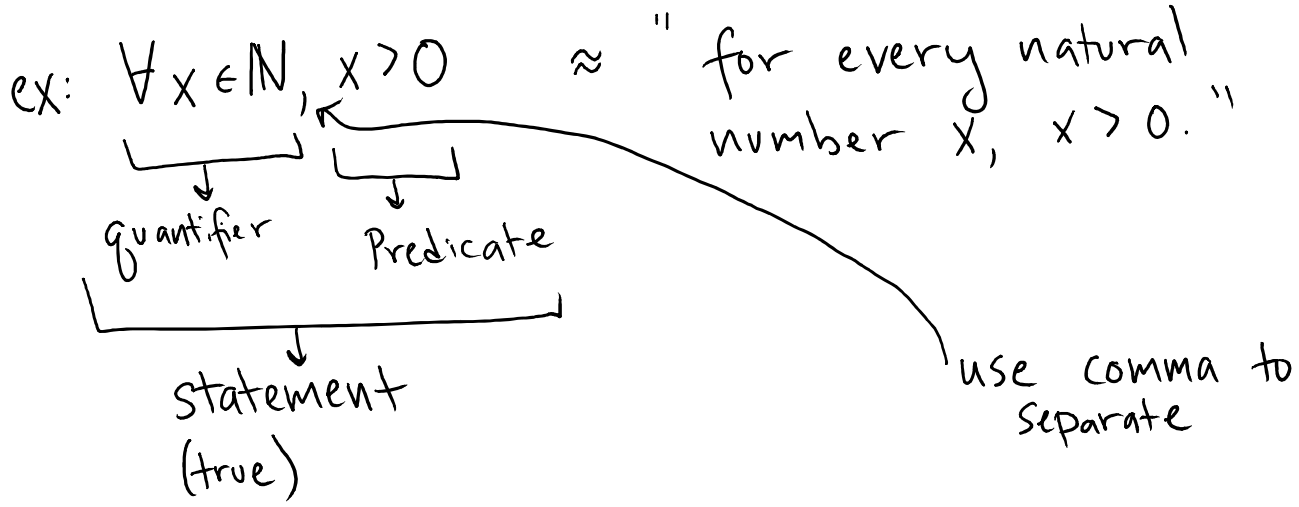


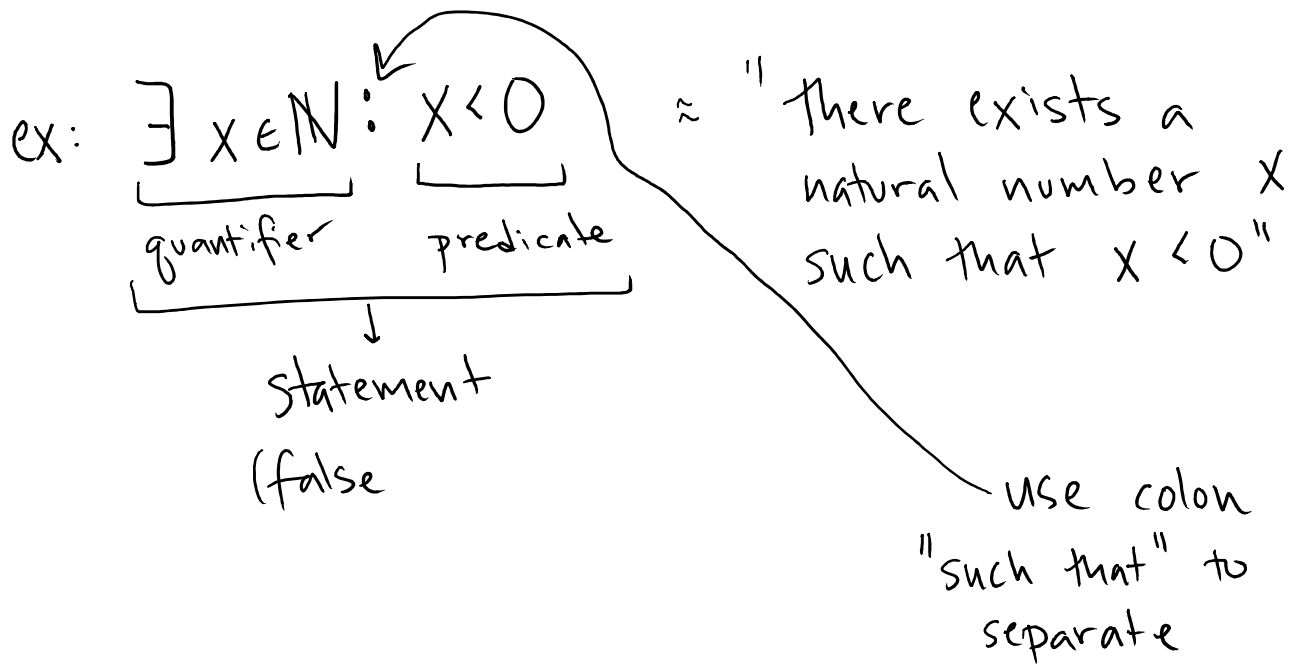
Goals: translate between English + Math

Quantifiers : turn predicates into statements

Universal Quantifier : \forall means "for all", "every"



Existential Quantifier : \exists means "there exists" "there is"



Q: Let $M(x, y)$ be the predicate that x is y 's parent. Let S be the set of all people who have ever lived.

True or false:

$$\forall x \in S, \exists y \in S: M(x, y)$$

Q: Let $M(x,y)$ be the predicate that x is y 's parent. Let S be the set of all people.

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False: It says for every person x , there is some person y such that x is y 's parent. In other words, it is saying every person is a parent, which is not true

How would you modify it to be true?

$$\forall x \in S, \exists y \in S: M(y,x)$$

This is saying every person has a parent.

* Quantifier must have domain

~~$$\forall x, x > 0$$~~

$$\forall x \in \mathbb{Z}, x > 0$$

domain = a set

Turning English Predicates Into Math:ex:

$P(n) \equiv$ n cents postage can be formed from 5¢ and 8¢ stamps

$$P(n) \equiv \exists x, y \in \mathbb{Z} : x, y \geq 0 \wedge 5x + 8y = n$$

(common construction $\exists m : m \wedge \neg m$)

Q: $m|n$ means m divides n . What is $2|6$?

- A) True B) False C) 3 D) $1/3$

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2 divides 6

ex:

 $E(n) = n$ is even $K(n) =$ all even integers are divisible by 2.

Want:

$$K(n) \equiv \forall x \in \text{Set of even #'s} \leftarrow \text{don't have this}$$

Instead:

$$K(n) \equiv \underbrace{\forall x \in \mathbb{Z}, E(n)}_{\text{does same thing}} \rightarrow 2 \mid n$$
(common construction: $\forall m, m \rightarrow m$)

Why? When writing proofs, usually easier to manipulate math. Useful to be able to convert from English to math

Important

- For input to Predicate NO QUANTIFIER
- For any other variable NEED QUANTIFIER

$\rightarrow S \equiv [\quad] \leftarrow$ all variables quantified
statement

x is quantified if write $\forall x \in _$, or $\exists x \in _$: