S. KIMMEL

Goals

- Familiarity with language and style of proofs.

Reflections

- $P(k) \to P(k+1)$ <u>NOT</u> $P(k+1) \to P(k)$
- How I go about inequality proof

Let S be the set of living people.

For $x \in S$, let $A(x)$ be the age of person x.

$$\forall x \in S,\ \exists y \in S:\ A(x) > A(y)$$

Q: What is the English translation?

A) There is an oldest person.

B) There is a youngest person.

C) There is no oldest person

D) There is no youngest person ←

CS200 - Worksheet 2

(Taken from *Discrete Mathematics, an Open Introduction* by Levin). For each of the following proofs, translate the statement that is proved into math. Use $m|n$ to be the predicate m divides n. Next discuss the language. What words are used repeatedly, and what do those words signal to the reader? What do you notice about the style? Anything else you notice?

1. Suppose a and b are odd. That is, $a = 2k + 1$ and $b = 2m + 1$ for some integers k and m. Then

$$\begin{aligned} ab &= (2k+1)(2m+1) \\ &= 4km + 2k + 2m + 1 \\ &= 2(2km + k + m) + 1. \end{aligned} \tag{1}$$

   Therefore, ab is odd.

2. Assume that a or b is even. Suppose it is a, since the case where b is even will be identical. That is, $a = 2k$ for some integer k. Then

$$ab = (2k)b = 2(kb). \tag{2}$$

   Therefore ab is even.

3. Suppose that ab is even but a and b are both odd. Namely, $a = 2k + 1$ and $b = 2j + 1$ for some integers k, and j. Then

$$\begin{aligned} ab &= (2k+1)(2j+1) \\ &= 4kj + 2k + 2j + 1 \\ &= 2(2kj + k + j) + 1. \end{aligned} \tag{3}$$

   But this means that ab is odd, which contradicts our premise. Thus a and b can not both be odd.

4. Assume ab is even. Namely, $ab = 2n$ for some integer n. Then there are two cases: a must be either even or odd. If it is odd, then $a = 2k + 1$ for some integer k. Then we have

$$\begin{aligned} 2n &= (2k+1)b \\ &= 2kb + b. \end{aligned} \tag{4}$$

   Subtracting $2kb$ from both sides, we get

$$2(n - kb) = b. \tag{5}$$

   Therefore, b must be even. The other case is that a is even, so we find that either a or b is even.