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Programming Assignment!

- · Use de Morgan's other laws
- · Prove using direct + contrapositive approaches

- Q: Which is logically equivalent to:

 \[\frac{1}{2} \times \times \times \]
- $A: \forall x \in \mathbb{Z}, x^2 + x \vee x \leq 1$
- B: YXEZ, X=XXXX=1
- C: $\exists x \in \mathbb{Z}$: $X^2 \neq x \vee x \leq 1$
- D: 3x EZ: X2 + X V X E 1
- Q: Which is logically equivalent to $7 \forall x \in \mathbb{Z}: 2|x \vee 3|x$
- A: ∀ X ∈ Z, 2+x
- B: YXEZ, 2+X / 3+X
- C: 3x eZ: 2+x V3+x
- D: 3x EZ: 2tx A 3+x
- 2+x = 72|x

Q: Which is logically equivalent to:

7 JXEZ: X2=X / X>1

A: V XEZ, X+XV XS

B: YXEZ, X3XXX X = 1

C: $\exists x \in \mathbb{Z}$: $X^2 \neq x \vee x \leq 1$

D: 3xeZ: x2 + x V x = 1

De Morgan's Law:

Q: Which is logically equivalent to:

7 \ X ∈ Z: 2 | X V 3 | X

A: YXEZ, 2tx V2tx

B: YXEZ, 2+X / 3+X

C: 3x \ Z: 2+x V 3+x

D: 3x EZ: 2tx 1 3+x

De Morgans Other Rule:

 $\neg (P \land Q) = (\neg P \lor \neg Q)$ $\neg (PVQ) = (\neg P \land \neg Q)$

· Distribute 7

· Switch / > V

De Morgan again

 $\Rightarrow \forall x \in \mathbb{Z}, (\neg x^2 = x \vee \neg x)$

=> \forall \times \in \bigg[\chi^2 = \times \forall \times \]

Rewrite

|> \forall \times \in \bigg[\times \forall \times \tim

De Morgan's Law

 $\exists X \in \mathbb{Z} : \neg (2|X \vee 3|X)$

De Morgan Again

=) $\exists \times \in \mathbb{Z}: (2 + \times \wedge 3 + \times)$

SKIMMEL

Direct Proof

Use: Prove P > Q

Structure:

Assume P. Explain, explain, explain. Therefore Q.

Use: Prove $\forall_X \in S$, $P(x) \rightarrow Q(x)$

Structure

Let XES, and assume P(x).

Explain, explain, explain.

Therefore Q(x)

If domain of X is obvious, can leave off
"Let XES," like proofs on worksheet. But it is
clearer if write it. So worksheet proof should have
"Let a, b EZ".

S.KIMMEL

You've already written a direct proof! Inductive step: $\forall k \in \mathbb{Z}$, $(k \ge base \land P(k)) \rightarrow P(k+1)$

Let K2 (base case). Assume for induction P(K) is true. Explain

Therefore, P(X+1) is true

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Q: Prove: If a|b and b|c then a|c.
(a|b = Je \in Z: ae=b)

Q: Prove: If a|b and b|c then a|c. $(a|b = \exists e \in \mathbb{Z}: ae=b)$

Let $a,b,c\in\mathbb{Z}$. Assume a|b and b|c. This means $f=a,f\in\mathbb{Z}$ such that b=ae and c=bf. Then $c=bf=(ae)f=\alpha(ef)$.

That means a/c, since c=ak, for k=ef, an integer.

Proof By Contrapositive

Structure:

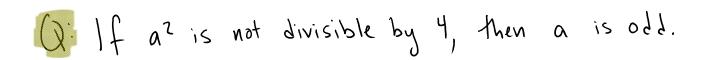
We prove the contrapositive. Assume 7Q. Explain explain, explain. Therefore, 7P.

· Use: Prove
$$\forall x \in S, P(x) \rightarrow Q(x)$$

Structure

We prove the contrapositive Let XES and assume TQ(X)

Explain, explain, explain.
Therefore, 7P(x).





Q: If az is not divisible by 4, then a is odd.

We prove the contrapositive. Let a & Z. Suppose a is even. Then $\exists k \in \mathbb{Z}: 2k=a$. This means $a^2 = 4k^2$. Since K2 is an integer, 4/a2.

There is an implied "for all" in the proof statement. (otherwise it is a predicate & we can't prove true or false). Therefore, we need "Let a & Z."

Iff = "if and only if.

Use: P -> Q

$$(P \rightarrow Q) \wedge (Q \rightarrow P) \equiv P \leftrightarrow Q$$

(use truth table proof)

Structure

P For the forward direction, [Proof of P-) Q]

P For the backwards direction, [Proof of Q->P]

Could be direct or contrapositive