

Goals

- Describe the connection between counting and probability
- Describe sample space, events, probability of an event
- Determine the probability of events.

Midterm

- Everything except probability (up to PS 8)
- Post to Canvas Discussion tonight to influence Wed. review
- Same system as first midterm

Q: If 8 people from a basketball team show up to a game, how many ways are there to form a 5 person team?

- A) 40      B) 56      C) 60      D) 112

Q:

How many DNA strings of length 5, i.e. strings in  $\{C, T, G, A\}^5$  have exactly 3 C's, and no other repeated letters?

Q: IF 8 people from a basketball team show up to a game, how many ways are there to form a 5 person team?

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$$\frac{8!}{5! \cdot 3!} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 8 \cdot 7 = 56$$

How many DNA strings of length 5, i.e. strings in  $\{C, T, G, A\}^5$  have exactly 3 C's, and no other repeated letters?

Task 1  
Choose where to put C's:

$$\binom{5}{2} = \frac{5!}{2! \cdot 3!} = \frac{5 \cdot 4}{2!} = 10$$

↑

Choose 2 out of 5 positions to have C's

and Task 2 = 60

Choose 2 of remaining 3 letters to put in remaining 2 spots (order matters)

$$P(3, 2) = \frac{3!}{1!} = 6$$

# Probability

Probability is all about counting

ex: What is the probability that the outcome of a die roll is at least 5?

1. Count all outcomes

$$|\{1, 2, 3, 4, 5, 6\}| = 6$$

2. Count outcomes where rolled value is  $\geq 5$

$$|\{5, 6\}| = 2$$

3. Take ratio:

$$\frac{2}{6} = \frac{1}{3}$$

## Terminology

- Sample space: set of all possible outcomes
- Event: subset of sample space.

← think big!

def: If all elements in sample space  $S$  are equally likely, the probability of an event  $E$  is

$$\Pr(E) = \frac{|E|}{|S|}$$

Note: Probability that an event  $E$  does NOT happen

$$1 - p(E)$$

★ Sometimes easier to calculate the probability of an event not happening, and use this rule to find the probability of an event happening. ★

Q: Lottery where a 4-digit number is chosen randomly. What is the size of the sample space?

- A) 40      B) 10,000      C)  $\binom{10}{4}$

↑

Use product rule!

$$10 \cdot 10 \cdot 10 \cdot 10$$

Q. Suppose you win some money if you get 3 of 4 numbers matching.

ex: Lotto: 5 7 0 1

You: 5 7 5 1

What is the probability you get 3 out of 4 correct?

- A)  $3/10^4$       B)  $10/10^4$       C)  $36/10^4$       D)  $40/10^4$

↑

Sum rule:  $9 + 9 + 9 + 9 = 36$

or Product rule:  $\binom{4}{3} \cdot 9 = 36$

Q. What is the probability that you DON'T win lottery?  
 (Don't get 3 or 4 digits the same)

A) 0.9962      B) 0.9963      C) 0.9964      D) 0.9965

↑

Sum rule → 
$$\frac{36 \text{ ways to match 3 #'s} + 1 \text{ way to match 4 #'s}}{37 \text{ ways to win}}$$

$$\Pr(\text{NOT win}) = 1 - \frac{37}{10000} = 0.9963$$