CS200 - Problem Set 6

1. Consider making postage out of 3-cent stamps and 7-cent stamps. You can always make postage as long as n is greater than or equal to some value n^*. You should figure out what n^* is, and then prove the result twice.

   (a) **[11 points]** Prove that you can make any postage greater than or equal to n^* cents using 3-cent stamps and 7-cent stamps using regular induction and proof by cases, as in the example from class.

   (b) **[11 points]** Prove that you can make any postage greater than or equal to n^* cents using 3-cent stamps and 7-cent stamps using strong induction. Please think a bit about how to do this on your own, but see hint on last page if you are getting stuck.

2. **[11 points]** Suppose $a, b, c \in \mathbb{Z}$. Then prove using a proof by contradiction that if $a^2+b^2 = c^2$, then a or b is even. (See hint on last page if stuck getting started.)

3. **[3 points]** When we use a direct proof to prove $P \to Q$ is true, we start by assuming P is true. Why do we not also consider the case that P is false?

4. For each of the following relations, either prove it is an equivalence relation or prove it is not an equivalence relation. If it is an equivalence relation, describe what the equivalence classes are (for an additional 6 points)

   (a) **[11 +?6 points]** Let S be the set of strings with more than n characters, and consider the relation R on S such that $(s, t) \in R$ if the first n characters of s are the same as the first n characters of t. (This relation is important for programming languages. For example, in the C programming language, the compiler only looks at the first 31 characters of a variable name.)

   (b) **[11 +?6 points]** Let $R \subseteq \mathbb{R} \times \mathbb{R}$. where $(a, b) \in R$ if and only if $|a - b| \leq 1$.

5. For a graph $G = (V, E)$, let $K(m, n, G)$ be the predicate: $\exists S, T \subset V : (|S| = m) \wedge (|T| = n) \wedge (S \cap T = \emptyset) \wedge (E = \{\{a, b\} : a \in S \wedge b \in T\})$. If a graph satisfies this predicate, we call the graph $K_{m,n}$

   (a) **[3 points]** Draw $K_{2,3}$

   (b) **[6 points]** Describe the graph $K_{1,n}$ in English.

Hint: to prove the stamp result by strong induction, consider the following. If $P(k)$ is true, the $P(k+3)$ is true, because you can just add a 3-cent stamp to your old solution. (In other words, if there is a solution 3 rungs down the ladder, there is a solution to the current rung.) You will need at least 3 base cases.

Hint: To prove a $P \to Q$ statement by contradiction, we assume $\neg(P \to Q)$, which is equivalent to $P \wedge \neg Q$.