CS200 - Problem Set 4

Please read the sections of the syllabus on problem sets and honor code before starting this homework.

1. **[6 points]** What can you deduce from the following true statements? Please write the simplest new true statement possible. (Partial credit will be given for more complex statements.)

$$\begin{array}{l} P \rightarrow R \\ Q \rightarrow R \\ P \vee Q \\ \hline \therefore \end{array} \tag{1}$$

2. (a) **[6 points]** Write the sentence "Children are always younger than their parents," where S is the set of all people who have every lived, $M(x, y)$ is the predicate x is the parent of y, and $A(x)$ is the age of person x.

   (b) **[6 points]** Now use de Morgan's rule to write a sentence using math that is logically equivalent to "Children are always younger than their parents," but that uses a different quantifier from what you previously used. (If you used $\exists$ above, you should use $\forall$ here, or vice versa. Note that $\neg(P \rightarrow Q) \equiv P \wedge \neg Q$.)

   (c) **[6 points]** Translate your sentence from part (b) back into English. Write it in as natural a way as possible.

3. Read section 5.3 of Proof by Richard Hammack. Then read the following poorly written proof of the statement: If n is even, then n^2 is even.

   Proof:

   1. Let $n =$ an integer.
   2. Suppose n is even.
   3. Then $n = 2k$.
   4. $n^2 = (2k)^2$, $(2k)^2 = 4k^2$, so $4k^2 = 2(2k^2)$
   5. Since $(2k^2)$ is an integer, I've shown it is even.

   (The sentences in the proof are numbered to make it easier to reference specific lines in your answer.)

(a) **[1 point per guideline violation found]** Identify sentences that violate Hammack's mathematical writing guidelines and explain why. (A sentence can violate multiple guidelines, and so can be included multiple times.)

(b) **[6 points]** Rewrite the proof so that it follows Hammack's mathematical writing guidelines.

4. **[11 points]** The pigeonhole principle is an extremely important tool in computer science (see this StackExchange post for just some of its many diverse applications). It states: If you put at least $n+1$ pigeons in n cubbies, there must be a cubby with more than one pigeon in it. Write a proof using the contrapositive

5. **[11 points]** Prove $\forall n \in \mathbb{Z}$, n is even if and only if $5n+3$ is odd. Prove one direction using a direct proof, and one direction using a contrapositive proof.

6. **[11 points]**

   Prove that Algorithm 1 correctly searches an array of integers for a specific integer.

   Input : Integer s, and an array of integers A

   Output: Returns i such that $A[i] = s$, or -1, if s is not in the array. (The first element of A is at position 1.)

```
l = length of A;
if A[l] = s then
    return l;
else
    if l == 1 then
        return −1;
    else
        return Search(s, A[1 : l − 1])
    end
end
```

Algorithm 1: `Search`(s, A)

7. **[0 points - for extra practice if you want it]** Let S be the set of all people who have ever lived. Let $A(x)$ be the age of person x. Let $M(x, y)$ be the predicate that x is y's parent. Please translate the following into either math or English as appropriate:

   (a) $\exists y, z \in S : M(x, y) \wedge M(y, z)$

   (b) There are two people who have the exact same age.

   (c) There are n people who have age a. (Use set cardinality and set-builder notation.)

   (d) $\forall x \in S, (A(x) > 90 \text{ years}) \rightarrow (\exists y \in S : M(x, y))$

8. **[Challenge]**
You meet two spiders on the road. Everyone knows that a spider either always tells the truth, or always lies. The first spider says, "If we are brothers, then we are both liars." The second spider says, "We are cousins or we are both liars." Are both spiders telling the truth? (Hint, create a truth table for their statements and consider the possible cases of each spider lying or telling the truth, and use deduction to see if there is a contradiction. Also, can brothers be cousins?)

9. How long did you spend on this homework?