CS200 - Midterm2

1. This exam is closed book, closed notes, closed internet, and closed person EXCEPT for a one-sided, hand-written 8.5×11 piece of paper. Please turn this in with your exam if you use it.

2. You have 3 hours to take the exam.

3. You have 3 hours 30 minutes from the time you pick up your exam from the box next to my office door (MBH 635) to the time you should return it to my office (slide it under the door if I am not around). You should sign in the time that you pick up the exam on the sheet on my door.

4. You may use extra scratch paper, but if you turn in additional paper please make sure it is clear what is scratch work and what is your solution.

5. Exams are available starting Wednesday, April 17 at 10am, and the latest you should turn an exam in is Friday, April 19 at 6pm.

Not on exam:

- Logarithms
- Graph Search

More practice

- Recommended textbooks!

Options

- Equivalence Relations
- Graph Pseudocode + complexity
- Big-O proofs
- Counting

Equivalence Relations

Decide if equivalence relation. If yes, what are equivalence classes? (S = set of all people who ever lived). If not, prove it.

- $\{(a, b)$: a, b have the same biological parents$\} \subseteq S \times S$
- $\{(a, b)$: a, b share a parent$\} \subseteq S \times S$

Pseudocode

- Write code for adjacency list and adjacency matrix that tests if there are any self-loops (edges from a vertex to itself).
- What is the runtime of each? Do a detailed and worst-case analysis

Big-O Proofs:

Prove $4x^2 + x - 1$ is $O(x^2)$

Prove $4x^2 + x - 1$ is not $O(x)$

Calculate the runtime:

Algorithm 1: `Swapping`(A)

Input : Array A of integers of length n
Output:

```
1  k = n − 1;
2  while k ≥ 1 do
3  |   for j = 1 to k do
4  |   |   if A[j] > A[j + 1] then
5  |   |   |   Swap A[j] and A[j + 1];
6  |   |   end
7  |   end
8  |   k = k − 1;
9  end
10 return A;
```

Counting

- Suppose you are creating a password that is 6 characters long, using numbers, capital letters, and lower case letters. How many passwords are possible, if you use 2 numbers, 2 lowercase letters, and 2 capital letters?

Equivalence Relations

- $\{(a, b)$: a, b have the same biological parents$\} \subseteq S \times S$
  - Equivalence relation! An example of an equivalence class is an only child. Another example is a set of siblings.
  - Reflexive: share same parents as yourself.
  - Symmetric: if a has the same biological parents as b, b has the same parents as a.
  - Transitive: if a has the same biological parents as b, and b has the same biological parents as c, then a has the same biological parents as c
- $\{(a, b)$: a, b share a biological parent$\} \subseteq S \times S$
  - Not transitive. Think about half and step siblings.

Pseudocode

- Write code for adjacency list and adjacency matrix that returns true if there are any self-loops (edges from a vertex to itself).
- What is the runtime of each? Do a detailed and worst-case analysis

Adjacency Matrix: A for G=(V,E)
For($v \in V$)
    If($A[v, v] = 1$): return True

Adjacency List: A for G=(V,E)
For($v \in V$)
    For($i \in A[v]$)
        If($i = v$): return True

Big-O Proofs:

Prove $4x^2 + x - 1$ is $O(x^2)$

- For $x > 1, x^2 \geq 1$. Also, $-1 < 0$, so for $x \geq 1$,

$$4x^2 + x - 1 \leq 4x^2 + x^2 = 5x^2.$$

Thus with $k = 1$ and $C = 5, 4x^2 + x - 1$ is $O(x^2)$

Prove $4x^2 + x - 1$ is not $O(x)$

- Assume for contradiction $\exists C, k: 4x^2 + x - 1 < Cx$ for all $x > k$. We can always choose $k \geq 1$, in which case, $4x^2 \leq 4x^2 + x - 1 < Cx$. Dividing by x (which we can do b/c $x \geq 1$ by our choice of k) and 4, we get $x < C/4$ which contradicts our claim that the inequality is true for all $x > k$.

Calculate the runtime:

Algorithm 1: Swapping(A)

Input : Array A of integers of length n

Output:

```
1  k = n − 1;
2  while k ≥ 1 do
3  |   for j = 1 to k do
4  |   |   if A[j] > A[j + 1] then
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6  |   |   end
7  |   end
8  |   k = k − 1;
9  end
10 return A;
```

$$\sum_{k=1}^{n-l} [ops\ in\ inner\ loop]$$

$$= \sum_{k=1}^{n-1} \left[\sum_{j=1}^{k} 1 \right]$$

$$= \sum_{k=1}^{n-1} [k]$$

$$= \frac{(n-1)n}{2} = O(n^2)$$

Counting

- Suppose you are creating a password that is 6 characters long, using numbers, capital letters, and lower case letters. How many passwords are possible, if you use 2 numbers, 2 lowercase letters, and 2 capital letters?
- Task 1, choose where to put numbers: $C(6,2)$
- Task 2, choose which two numbers to put: 10^2
- Task 3, choose where to put lowercase: $C(4,2)$
- Task 4, choose which two lower case letters: 26^2
- Task 5, choose which two capital letters: 26^2

Combine with product rule: $C(6,2) \times 10^2 \times C(4,2) \times 26^2 \times 26^2$