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Learning Goals

- Identify statements and predicates
- Create proofs using truth tables
- Deduce new truths

- Quiz on Canvas starting tomorrow
- Reflections: challenged, but learning
- Grace period of 1 hour from tutoring for pset turn in

Statements ≈ math sentences

also called proposition

definition: A statement is a declarative sentence that is true or false.

Q: Which are statements? Discuss

1. The majority of Middlebury students have 1 sibling.
2. The product of 2 and 5.
3. This sentence is not true.
4. $2 + x = 10$.

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Statements ≈ math sentences

also called proposition

def: A statement is a declarative sentence that is true or false.

Q: Which are statements? Discuss

1. The majority of Middlebury students have 1 sibling.
2. The product of 2 and 5. Not a sentence (no verb!)
3. This sentence is not true. Can't be true or false
4. $\boxed{2 + x = 10}$. Could be true or false, but now is neither!

A) All B) 1,3 C) 2,4 D) 1

Predicate - What is it? Becomes a statement if variable gets a value!

We've seen predicates in inductive proofs

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Proof Goal:

Known true statements $\rightarrow$ New true statements

One tool: combining statements

I learn. = "P"
I study. = "Q"

} assign letter to basic, atomic statements

English	Math	Python	Java
P and Q	$P \land Q$	and	&&
P or Q	$P \lor Q$	or	||
not P	$\neg P$	not	!
if P then Q	$P \rightarrow Q$		
P if and only if Q	$P \leftrightarrow Q$		

abbreviated P iff Q

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Truth Table

P	Q	$P \to Q$	$P \leftrightarrow Q$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	T

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Truth Table Proofs

Prove the following statement is true:

If you eat a raw egg everyday, then you will win the lottery, or if you win the lottery you will lose your job.

P

Q

R

1. Identify and label atomic statements
2. Write statement symbolically

$$(P \to Q) \vee (Q \to R)$$

3. Create truth table. Use it to show statement always true

Q: How many rows will the truth table have?

A) 3 B) 4 C) 6 D) 8

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ATOMIC

Build Up to Complex

P	Q	R	$P \to Q$	$Q \to R$	$(P \to Q) \vee (Q \to R)$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

8 Rows

all possible combinations

Proof: Let P be the statement . . .

" Q "

" R "

Then looking at the truth table, we see $(P \to Q) \vee (Q \to R)$ is true for any truth value of P, Q, and R.

Truth tables are only useful when you don't have too many atomic statements $\left(\text{\# of rows} \sim 2^{(\text{\# of atomic statements})}\right)$

Better if can avoid

Q. Explain why $(P \to Q) \vee (Q \to R)$ is true without a truth table

- Q is true or false. If it is true, $P \to Q$ is true. If it is false, $Q \to R$ is true. Either way, at least one of $P \to Q$, $Q \to R$ is true, so the whole statement is true.

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Useful tool: Logical Equivalences

ex: $P \to Q$ is logically equivalent to $(\neg P) \vee Q$

P	Q	$P \to Q$	$\neg P$	$\neg P \vee Q$	$(P \to Q) \leftrightarrow \neg P \vee Q$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Statements are logically equivalent $\iff$ Have same truth values for any assignment of atomic substatements

☆ If two statements are logically equivalent, can substitute one for the other.