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Learning Goals

- Describe Inductive Proofs at a high level
- Describe inductive proof structure
- Find errors in inductive proofs

Announcement

- Taken lots of math? Come see me.

Sample Syllabus Quiz Question:

Q: Which of the following problem set parts are graded for correctness?

A. Rough Draft.

B. Main PSet Submission

C. Self Grade

D. None of them

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Learning Goals

- Describe Inductive Proofs at a high level
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- Find errors in inductive proofs

Announcement

- Taken lots of math? Come see me.
- Prequiz due Today, Rough Draft Sat

Sample Syllabus Quiz Question:

Q: Which of the following problem set parts are graded for correctness?

A. Rough Draft.

B. Main PSet Submission

C. Self Grade

D. None of them ⟸ All graded on effort, although you will get the most out of self-grade if you try for accuracy.

Syllabus Discussion (at end)

- Problem Set: Rough Draft, PSET, Self-Grade, Reflection
  - Rough Draft: 1pt ← Demonstrate you have thought a little about PSET
  - PSET: 1pt
  - Self-Grade: 1pt
  - Reflection: 3pt

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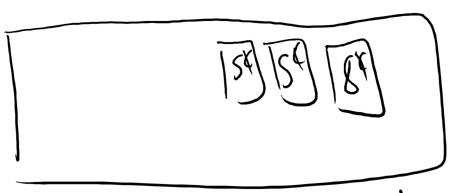
Induction

Recursive algorithm correctness $\longleftrightarrow$ Proof by induction

Example

Suppose you have unlimited 5¢ stamps and 8¢ stamps.
What postage values can you create?



18¢ ✓

What about 4¢? No!

What about 28¢? Yes!

What about 85,694¢? ???

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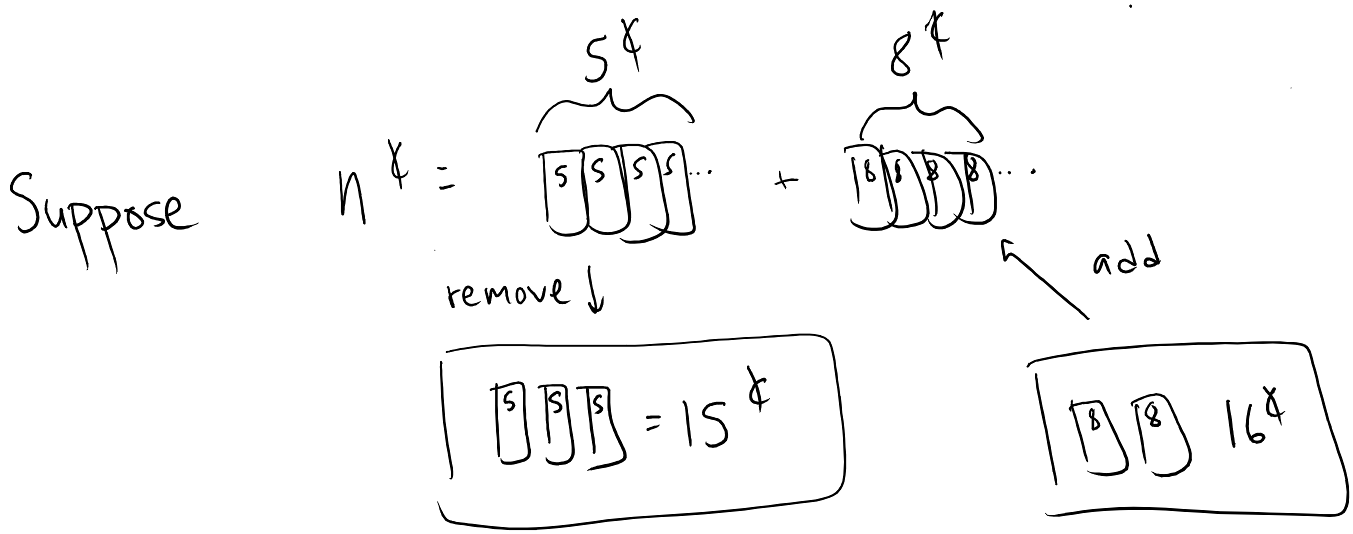
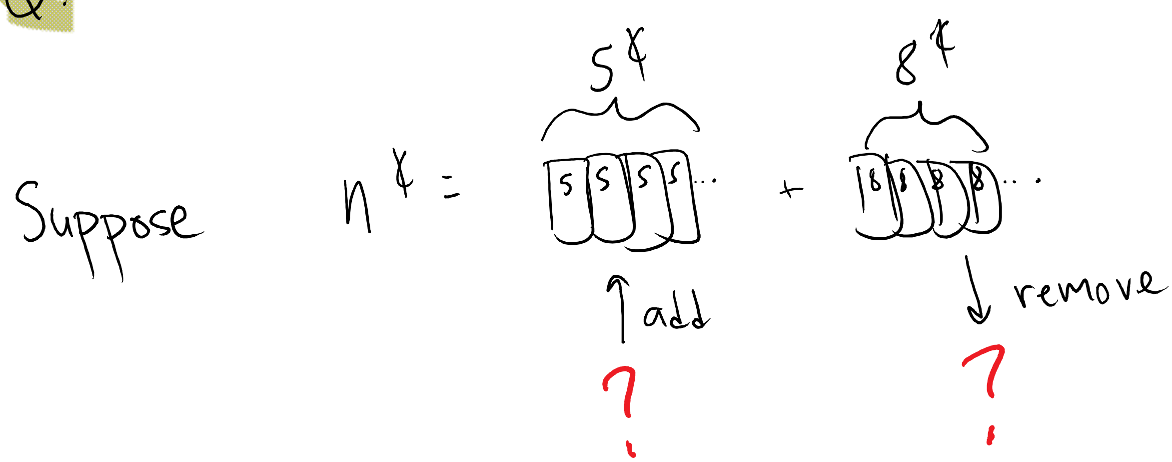
Induction: use old solution to get new solution



$n¢ \rightarrow (n+1)¢$

Q:



$n¢ \rightarrow (n+1)¢$

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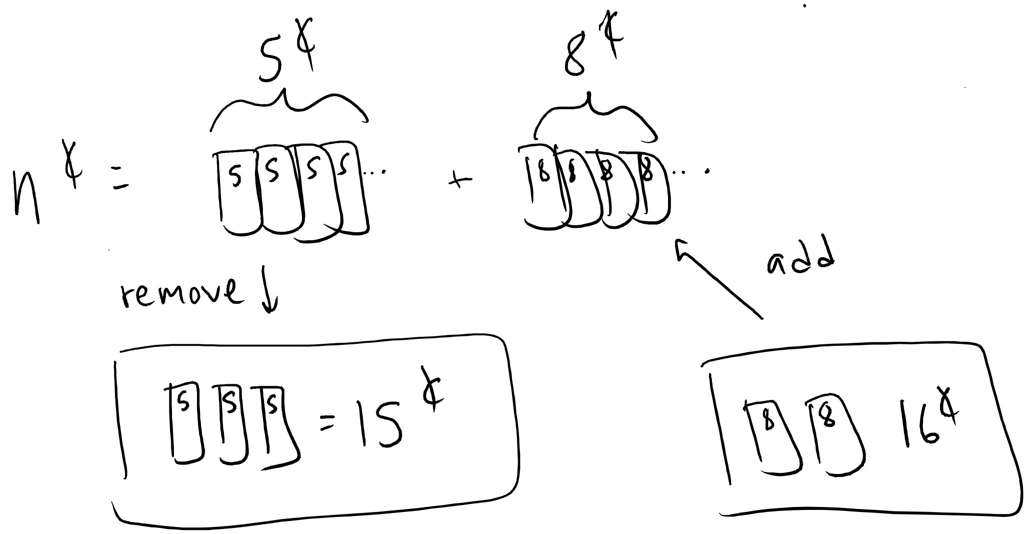
Induction: use old solution to get new solution

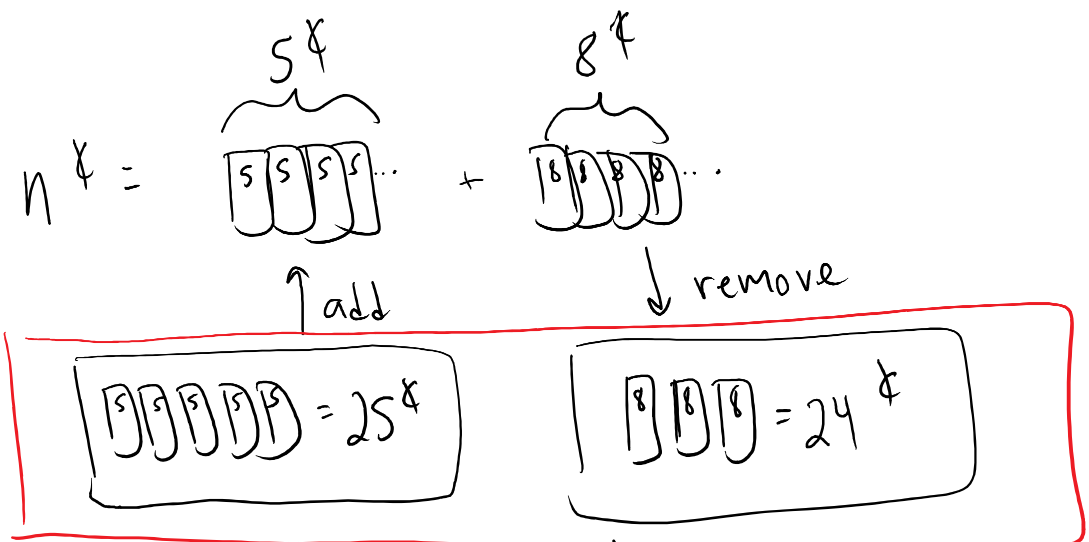
Suppose



$n¢ \to (n+1)¢$

Q:

Suppose



A:

$n¢ \to (n+1)¢$

Consequence: If can create $n¢$ with at least 3 [5] or at least 3 [8], can create $n+1$ ¢

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$28¢ = 4 \cdot \boxed{5} + 1 \cdot \boxed{8}$

$29¢ = 1 \cdot \boxed{5} + 3 \cdot \boxed{8}$

$30¢ = 6 \boxed{5}$

$31¢ = 3 \boxed{5} + 2 \cdot \boxed{8}$

$\vdots$

Q: If $85{,}693¢ =$

$5{,}761 \boxed{5} + 7111 \cdot \boxed{8}$

then can create

$85{,}694¢$ as

? $\_\_\_\_ \boxed{5} + \_\_\_\_ \boxed{8}$

or

$\_\_\_\_ \boxed{5} + \_\_\_\_ \boxed{8}$

A) 5759 / 7114

B) 5764 / 7108

C) 5766 / 7108

D) 5758 / 7113

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$28¢ = 4 \cdot \boxed{5} + 1 \cdot \boxed{8}$

$29¢ = 1 \cdot \boxed{5} + 3 \cdot \boxed{8}$

$30¢ = 6 \boxed{5}$

$31¢ = 3 \boxed{5} + 2 \cdot \boxed{8}$

$\vdots$

Q: If $85{,}693¢ =$

$5{,}761 \boxed{5} + 7111 \cdot \boxed{8}$

then can create

$85{,}694¢$ as

$\underline{5758} \boxed{5} + \underline{7113} \boxed{8}$

or

$\underline{5766} \boxed{5} + \underline{7108} \boxed{8}$

Answer:

A) 5759 / 7114

B) 5764 / 7108

C) 5766 / 7108 ←

D) 5758 / 7113 ←

find first solution, & the rest fall into place

$\Downarrow$

* Any postage $\geq 28¢$ is possible

Start at $28¢ \to 29¢ \to 30¢ \ldots \; 85{,}693¢ \ldots$

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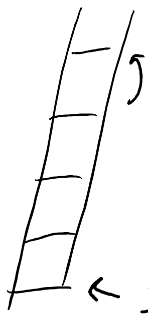
Principle of Induction: solution to smaller problem provides solution to larger problem

Stamps - need to have solution to n to get to n+1
Once you get 28¢ solution, we're good - always at least 3 5¢ or 8¢

Inductive Metaphor

Ladder



2. Show how to move from each rung to next

1. Show how to get on first rung (1st solution)

Shows you can get to all rungs! (Above 1st rung)

Formal Inductive Proof

Proofs have a unique style/language
- Essay vs. Texting vs News article vs. lab notebook

Different writing styles

This class → proof language.
Induction proof has a recipe, so easier style than other proofs.

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Inductive proof recipe:

(Set-Up)

sentence with a variable in it

Let P(n) be the predicate n¢ of postage can be formed from 5¢ and 8¢ stamps

We will prove, using induction on n, that P(n) is true for all $n \geq$ 28.

(Base Case)

Base case: P(28) is true because ______________

(Inductive Step)

base case #

Inductive case: Let $k \geq$ 28. Assume, for induction, that P(k) is true.

That means ______________

So ______________

Then ______________

Plugging in ______________

P(k)

P(k+1)

Don't go back ward

Thus P(k+1) is true

(Conclusion)

Therefore, by induction, P(n) is true for all $n \geq$ ___.

Q: Put the following sentences in the correct order, and identify and correct any errors.

Then there exists an integer b such that $7^k - 1 = 6b$.

Because b is an integer, $7b + 1$ is an integer, so $P(k + 1)$ is true.

Inductive Step: Let $k \geq 1$ and assume that $P(k)$ is true.

Let $P(n)$ be the predicate $7^n - 1$ is a multiple of 6 for all $n \geq 0$.

Base Case: $P(1)$ is true because $7^1 - 1 = 6$, which is a multiple of 6 since $6 \times 1 = 6$.

We will prove using induction that $P(n)$ is true.

Therefore, by induction on n, $P(n)$ is true for all $n \geq 0$.

Multiplying both sides by 7, we get $7^{k+1} - 1 = 6(7b + 1)$.

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Proof that $7^n - 1$ is a multiple of 6 for all $n \geq 0$, with errors corrected:

Let $P(n)$ be the predicate $7^n - 1$ is a multiple of 6. ~~for all $n \geq 0$.~~ We will prove $P(n)$ is true for all $n \geq 0$.

Why?

P is a function that takes in a number and outputs a sentence

- $P(n) \Rightarrow$ "$7^n - 1$ is a multiple of 6 for all $n \geq 0$."

  $P(2) \Rightarrow 7^2 - 1$ is a multiple of 6 $\underbrace{\text{for all } 2 \geq 0}_{\text{doesn't make sense}}$

We will prove using induction that $P(n)$ is true.

Base Case: ~~$P(1)$~~ $P(0)$ is true because ~~$7^1 - 1 =$~~ $7^0 - 1 = 0$ 6, which is a multiple of 6 since ~~$6 \times 1 = 6$~~. $6 \times 0 = 0$.

Inductive Step: Let ~~$k \geq 1$~~ $k \geq 0$ and assume that $P(k)$ is true.

Then there exists an integer b such that $7^k - 1 = 6b$.

Multiplying both sides by 7, and adding 6 to both sides we get $7^{k+1} - 1 = 6(7b + 1)$.

Because b is an integer, $7b + 1$ is an integer, so $P(k + 1)$ is true.

Therefore, by induction on n, $P(n)$ is true for all $n \geq 0$.