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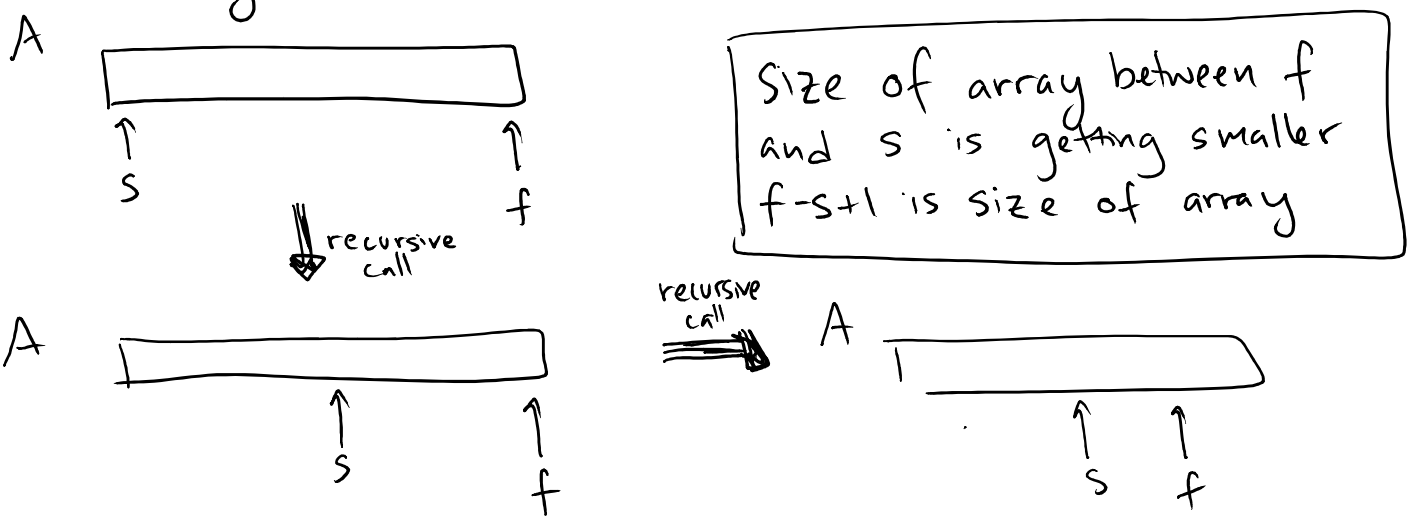
Goals

- Be able to describe functions + their properties using math language.
- Describe connection between relations + functions
- Analyze properties of functions

Binary Search

- Recursive algorithm: use induction / strong induction
- Look at how size of input decreases in recursive call.
  - Make this size your variable "n".
  - If size decreases by exactly 1 ⇒ use induction
  - Else ⇒ use strong induction

In Binary Search



Size of array between f and s is getting smaller
f-s+1 is size of array

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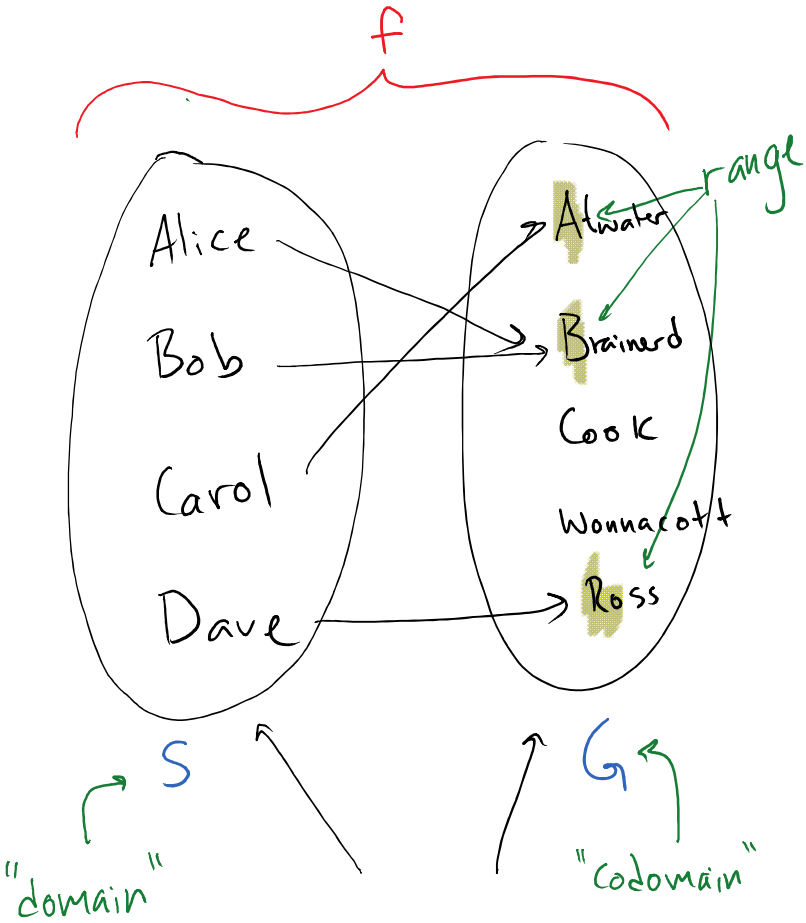
Functions: way to relate elements of 2 sets

ex: Commons affiliation



2 sets

You give the function a student name as input, it gives a grade as output

We write:

- $f: S \to G$

This arrow means something different than if-then. How to tell? Are there predicates or set?

input set — output set

means "f is a function from domain S to codomain G"

- $f(\text{Carol}) = \text{Atwater}$
  - Atwater is "image" of Carol
  - Carol is "preimage" of Atwater

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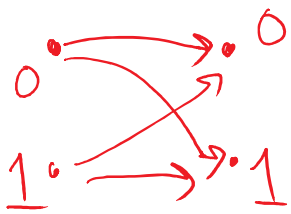
Q: Which is true?

A: Every relation on A can be expressed as a function $f: A \to A$

B: Every function $f: A \to A$ can be expressed as a relation on A.

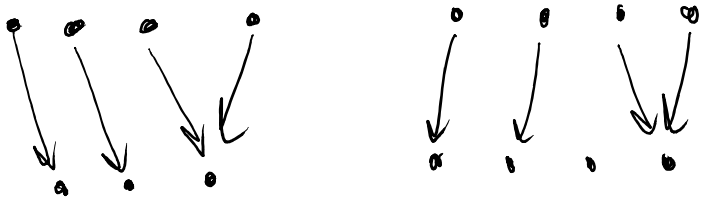
B: $R_f \subseteq S \times G$, $R = \{(s, g) : f(s) = g\}$

A is false. Ex $R = \{0,1\} \times \{0,1\} = \{(0,0), (0,1), (1,0), (1,1)\}$



3 important properties

Surjection "Onto"



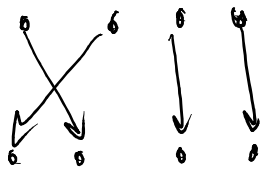
Surjective

Not Surjective

Injection "One-to-One"

Injective

Not injective



Injective & Surjective = Bijective

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Surjective: every codomain element has a preimage

Injective: no two elements of domain map to same element of co-domain

Q: Write using math:

- A function $f: S \to G$ is surjective if $\forall x \in G, \exists y \in S: f(y) = x$
- A function $f: S \to G$ is injective if $\sim \exists y_1, y_2 \in S: (y_1 \neq y_2) \wedge f(y_1) = f(y_2)$

Q: Is this function

A: Surjective
B: Injective
C: Bijective
D: None

S = set of Midd students
E = set of Midd e-mails

- $f: S \to E$, $f(x)$ = x's e-mail

  Injective: b/c no two ppl have the same email

- $f: S \to \{1, 2, \ldots, 12\}$, $f(x)$ = x's birth month

  Surjective: b/c some student is born in every month

- $f: S \to \mathbb{N}$, $f(x)$ = x's age in yrs

  None (no Midd students are 200 yrs old, 2 Midd students are same age)