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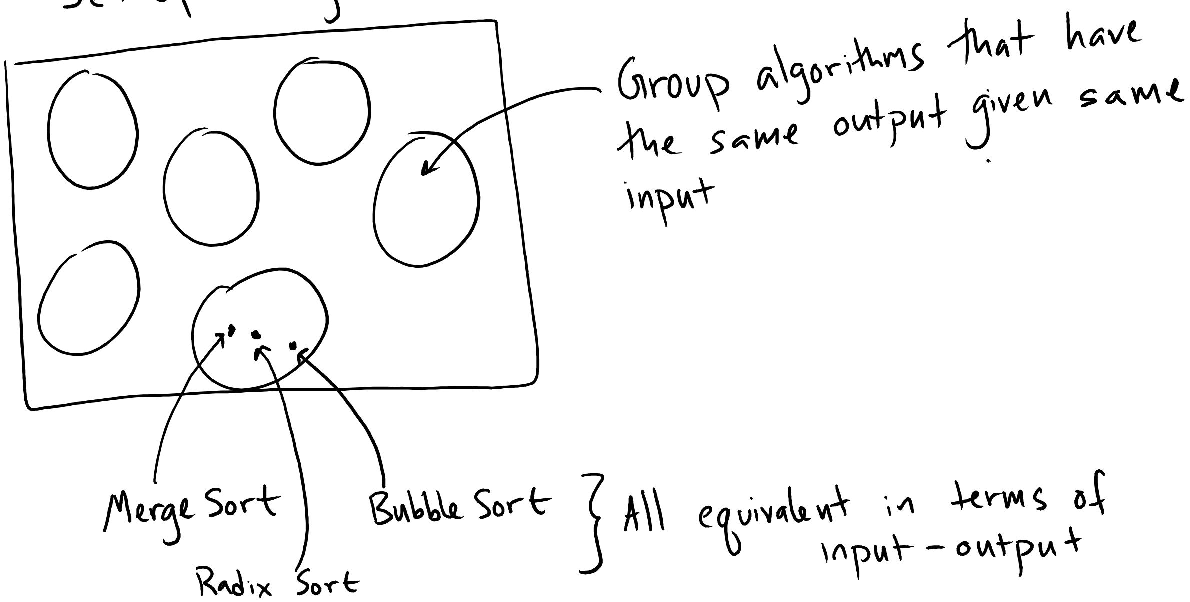
Goals

- Describe a relation
- Describe relationship between equivalence classes and relations.
- Prove whether a relation is an equivalence relation

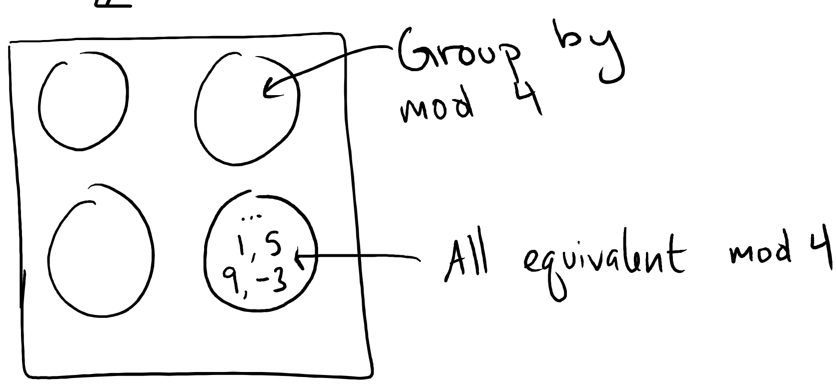
Equivalence Classes

Can be useful to divide elements of a set into subsets, and think of all elements in a subset as equivalent.

Example: Set of All Algorithms



Example: $\mathbb{Z}$



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We call these subsets <u>equivalence classes</u>.

How to define mathematically? Use <u>Relations</u>.

<u>def</u>: Let A be a set. Then a relation R on A is $R \subseteq \underbrace{A \times A}$

Cartesian Product

Subset

$(m \in R) \rightarrow (m \in A \times A)$

$\{(a,b) : a, b \in A\}$

regular parentheses means order matters

Q: Which of the following relations on $\mathbb{Z}$ has $(1,-1)$ as an element?

A) $\{(a,b) : a^2 = b\}$ ← contains $(-1,1)$

$\boxed{\text{B)}}$ $\{(a,b) : a = b^2\}$ ← $1 = (-1)^2$

Order matters

C) $\{(a,b) : ab = 1\}$

D) None of above

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Equivalence Relation:

def: $R \subseteq A \times A$, R reflexive, symmetric, transitive

$\forall a \in A, (a,a) \in R$

$\forall a,b \in A, (a,b) \in R \rightarrow (b,a) \in R$

$\forall a,b,c \in A, \big((a,b) \wedge (b,c)\big) \rightarrow (a,c)$

Equivalence Relations → Equivalence Classes:

$\boxed{(a,b) \in R}$ means $\boxed{a \equiv b}$ means $\boxed{\text{a,b in same equivalence class}}$

$\uparrow$ is equivalent to

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Q: What is an equivalence class for the equivalence relation $L = \{(a,b) : \text{length}(a) = \text{length}(b)\} \subseteq S \times S$

set of all bit strings

A) $(00, 11)$

B) $\{(0,0), (0,1), (1,0), (1,1)\}$

$\boxed{\text{C)}}$ $\{00, 01, 10, 11\}$

D) It's not an equivalence relation, so can't get equivalence classes

Equivalence classes are sets of bitstrings with the same length

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A) Equivalence Relation

B) Not reflexive

C) Not symmetric

D) Not transitive

1. $$R = \{(a,b) \in \mathbb{R} \times \mathbb{R} : a - b \in \mathbb{Z}\}$$

A) Equivalence Relation

- Reflexive: Let $a \in \mathbb{R}$. Then $a - a = 0$, and $0 \in \mathbb{Z}$, so $(a,a) \in R$.

- Symmetric: Let $a, b \in \mathbb{R}$. Assume $(a,b) \in R$. Then $a - b \in \mathbb{Z}$, so $b - a \in \mathbb{Z}$, so $(b,a) \in R$.

- Transitive: Let $a, b, c \in \mathbb{R}$. Assume $(a,b), (b,c) \in R$. Then $\exists\, x, y \in \mathbb{Z} : a - b = x \wedge b - c = y$. Thus $a - c = x + y$ which is an integer, so $(a,c) \in R$

$\Rightarrow$ Equivalence Classes: sets of numbers that have the same fractional part

ex: $\{\ldots, -1.9, -.9, 0.1, 1.1, 2.1, \ldots\}$