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Goals:
- Implement Deduction Strategies
- Create sets using set builder notation

$\boxed{\text{Quiz Reminder}}$

Deductions: Known true statements → new true statements

Q

If you graduate, you must pass a swimming test. $= T$

You graduated $= T$ ← P

Premises

∴ You passed a swim test. $= T$ Conclusion

$$\begin{array}{l} P \to Q \\ \underline{P} \\ \therefore Q \end{array}$$

If $P \to Q$ is true and P is true, Q must be true

2 Strategies

1. Truth table. Cross out false rows, see what is left

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

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2. Reason it out.

If P is true and $P \to Q$ is true then Q must be true because otherwise $T \to F = F$.

Q:

Andre has a black suit and a tweed suit. He always wears his tweed suit OR he wears sandals. If he wears his tweed suit and purple shirt, he does not wear a bow tie. He never wears his tweed suit unless he also wears a purple shirt OR sandals. If he wears sandals, he also wears a purple shirt. Yesterday, Andre wore a bow time. What else did he wear?

OR = logical or

W = tweed suit

P = purple shirt

S = sandals

B = bow tie

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W = tweed suit

P = purple shirt

S = sandals

B = bow tie

$\Rightarrow$

$W \vee S$

$W \wedge P \rightarrow \neg B$

$W \rightarrow (P \vee S)$

$S \rightarrow P$

B

$W \wedge P = F$

or

$W = F$

$\downarrow + W \vee S$

$S = T$

$\downarrow + S \rightarrow P$

$P = T$

$P = F$

$\downarrow + S \rightarrow P$

$S = F$

$\downarrow + W \vee S$

$W = T$

$\downarrow$ $W \rightarrow (P \vee S) = F$ ✗

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$W \vee S$

$W \wedge P \to \neg B$

$W \to (P \vee S)$

$S \to P$

B

$W \wedge P$ is false

Solution

W	P	S	B
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

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Sets

Set is a group of unordered objects (no repeats, order doesn't matter)

Metaphor: Folder $\overset{= \text{set}}{}$ on computer

- Contains files + other folders
  - files ↑ objects
  - folders ↑ sets

- Could be empty

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Roster Notation: list elements of set

$A = \{0, 2, 5\}$ means "A is the set containing the elements 0, 2, 5."

for sets "element" = "object"

$\in$: $2 \in A$ means 2 is an element of A

$\notin$: Prof. Watson $\notin S$ means Prof. Watson is not an element of S.

Sets in Sets: $T = \{x, y, \{g, h\}, k\}$

an element of a set can be another set

Q: Is $g \in T$? Is $\{g, h\} \in T$?

A) Yes. Yes. B) Yes. No. $\boxed{\text{C) No. Yes.}}$ D) No. No.

elements of T are x, y, $\{g,h\}$, k

*Also $\{x\ y\} \notin T$

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Famous Sets

$\emptyset$ = empty set = $\{\}$

$\mathbb{N}$ = set of natural numbers = $\{1, 2, 3, \dots\}$

$\mathbb{Z}$ = set of integers = $\{\dots -3, -2, -1, 0, 1, 2, 3 \dots\}$

$\mathbb{R}$ = set of real numbers

$\mathbb{Q}$ = set of rational numbers (fractions

$\boxed{\text{NOTE}}$: In some books, $\mathbb{N} = \{0, 1, 2, 3, \dots\}$

starts at 0.

Set Builder Notation

$B = \{ f(x) : P(x) \}$ = the set of x where P(x) is true, with f(x) applied to each element

function of x

predicate of x

ex: $A = \{x^2 : x \text{ is even}\} = \{0^2, 2^2, 4^2, 6^2, \dots\}$

$= \{0, 4, 16, 36, \dots\}$

$A = \{(2x)^2 : x \in \mathbb{Z}\} = \{(2\cdot 0)^2, (2\cdot 1)^2, (2\cdot 2)^2, \dots\}$

$= \{0, 4, 16, \dots\}$

$A = \{x : x \in \mathbb{N} \wedge \frac{\sqrt{x}}{2} \in \mathbb{Z}\}$