Section A

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Section B

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Review

- Sample space – set of possible outcomes, S
- Event – subset of S, E
- Probability of an event (if all outcomes equally likely): $Pr(E) = \frac{|E|}{|S|}$

If not all equally likely?

Assign a probability to each element of S:

$Pr: S \to \mathbb{R}$ such that

- $\forall\, i \in S, \quad 0 \leq Pr(i) \leq 1$
- $\sum_{i \in S} Pr(i) = 1$

Then the probability of $E \subseteq S$ is

$$Pr(E) = \sum_{i \in E} Pr(i)$$

means add up $Pr(i)$ for all elements $i \in E$

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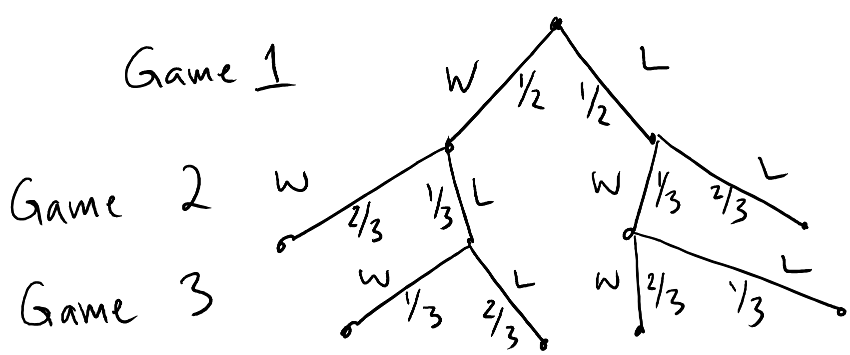
What happens if probability of one outcome depends on another?

ex: You are in a quidditch series against Skidmore. First team to win 2 games is champion

- Midd has 1/2 chance of winning first game
- If Midd won the previous game, has a 2/3 chance of winning next.
- If Midd lost the previous game, has a 1/3 chance of winning next.

What is the probability Midd wins?

Use tree:



Put probability on each edge

Outcomes are all paths from root to leaves:

$S = \{WW, WLW, WLL, LWW, LWL, LL\}$

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To calculate probability of each element in S, use conditional probability:

$$P(E \cap F) = P(E|F) \cdot P(F)$$

Probability both event E and event F occur

Probability of event E happening <u>if</u> you know event F happened

Probability event F occurs

$P(\cdot|\cdot)$ = "conditional probability"

<u>ex:</u>

P(Win game 1 + win game 2) =

P(win game 2 if won game 1) · P(win game 1)

$$= \frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3}$$

Can extend:

$$P(E \cap F \cap G) = P(E|F \cap G)P(F|G)P(G)$$

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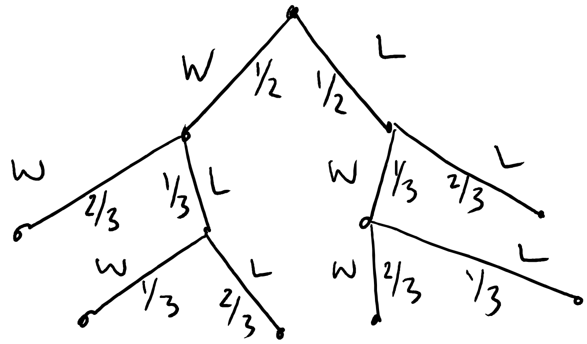
Game 1

Game 2

Game 3



Using conditional probability, probability of each outcome is product of probabilities on path

$$WW = \tfrac{1}{2} \cdot \tfrac{2}{3} = \tfrac{1}{3}$$
$$WLW = \tfrac{1}{2} \cdot \tfrac{1}{3} \cdot \tfrac{1}{3} = \frac{1}{18}$$
$$WLL = \tfrac{1}{2} \cdot \tfrac{1}{3} \cdot \tfrac{2}{3} = \frac{1}{9}$$

Event Midd Wins = $\{WW, WLW, LWW\}$

$$\Pr(\text{Midd Wins}) = \Pr(WW) + \Pr(WLW) + \Pr(LWW)$$
$$= \tfrac{1}{3} + \tfrac{1}{18} + \tfrac{1}{9} = \tfrac{1}{2}$$

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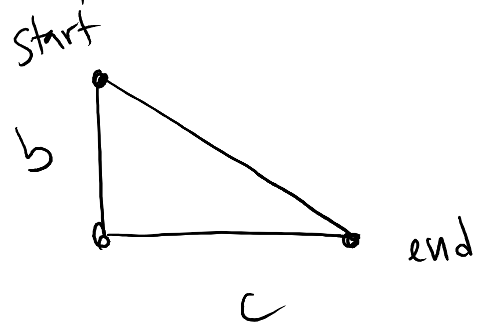
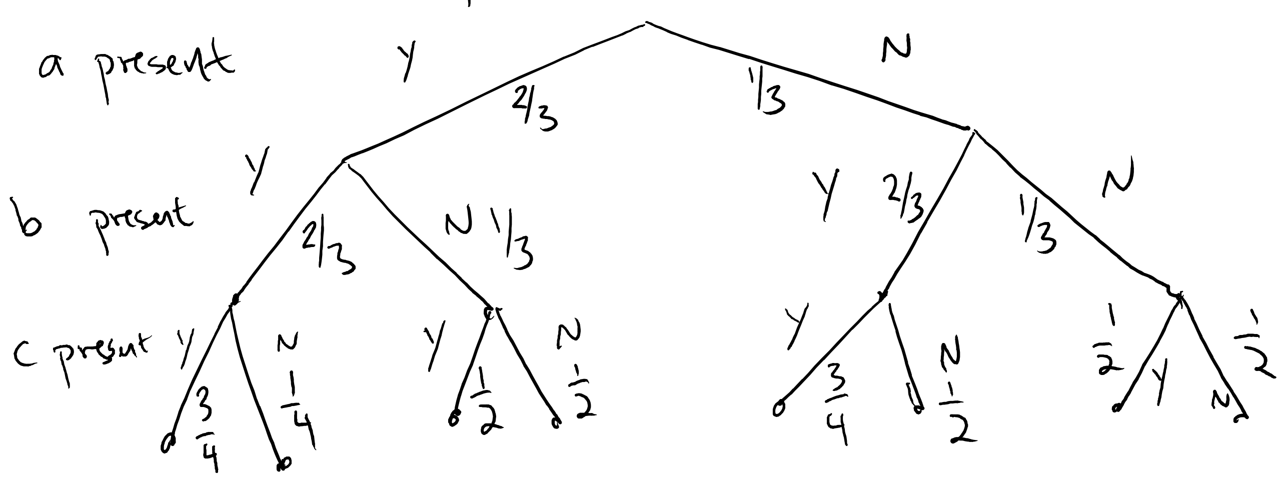
Group Work



- a, b present with prob $2/3$
- c present with prob $3/4$ if b present, otherwise $1/2$

What is the probability of percolation?



$$E = YYY, YYN, YNY, YNN, NYY$$

$$= \frac{1}{3} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{6} = 0.722$$

(Once you get comfortable, can do without tree)

$$YYN = P(Y_a) \cdot P(Y_b \mid Y_a) \cdot P(N_c \mid Y_a \text{ and } Y_b)$$