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Q: Prove it takes $n-1$ breaks to reduce an n-square chocolate bar to n individual squares.

A: Let $P(n)$ be the predicate " ." We will prove via strong induction that $P(n)$ is true for $n \in \mathbb{N}$, $n \geq 1$.

Base case: When you have a 1-square chocolate bar, it requires 0 breaks to create 1 individual squares, so $P(1)$ is true.

Inductive Case: We assume for strong induction that $P(i)$ is true for $1 \leq i \leq k$. We will prove $P(k+1)$ is true. Since $k+1 > 1$, we can break the chocolate into two pieces, one with a squares, and one with b squares, where $a+b = k+1$, and $1 \leq a \leq k$, and $1 \leq b \leq k$. Using our inductive assumption, it requires $(a-1)$ breaks to separate the first piece and $(b-1)$ breaks to separate the second. Adding up all the breaks, we have

$$(a-1) + (b-1) + 1 = a+b-1 = k$$

total breaks. Thus $P(k+1)$ is true.

Therefore, by strong induction, $P(n)$ is true.

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F(n)
1. If $n \leq 1$: return n
2. return $5 \cdot F(n-1) - 6 \cdot F(n-2)$

Let P(n) be the predicate F(n) returns $3^n - 2^n$. We will prove P(n) is true for all $n \geq 0$, using strong induction

Base case: There are two base cases: when the input is 0, we return 0. Since $3^0 - 2^0 = 1 - 1 = 0$, this is correct. When the input is 1, we return 1. Since $3^1 - 2^1 = 3 - 2 = 1$, this is correct.

Inductive step: We assume P(K) is true $\forall K \in \mathbb{Z} : 1 \leq K \leq k$. No consider input k+1. Now $k+1 \geq 2$, so we return $5F(k) - 6F(k-1)$. Since $k \geq 1$, F(k) returns $3^k - 2^k$ by inductive assumption. Since $k-1 \geq 0$, and F(0) is correct by the base case, and larger values are true by inductive assumption, F(k-1) returns $3^{k-1} - 2^{k-1}$. Thus the function returns

$$
\begin{aligned}
&5(3^k - 2^k) - 6(3^{k-1} - 2^{k-1}) \\
&= 5 \cdot 3^k - 5 \cdot 2^k - 2 \cdot 3^k + 3 \cdot 2^k \\
&= 3^{k+1} - 2^{k+1}
\end{aligned}
$$

as desired.

Therefore, by strong induction, F(n) is correct.

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Therefore, by strong induction, $P(n)$ is true for $n \in \mathbb{N}$, $n \geq 1$.

Multiple Base Cases:

To prove $P(n)$ maybe only need to assume $P(n-1)$ and $P(n-2)$ are true. Then to prove $P(3)$, need $P(2)$ and $P(1)$ true

Prove both
in base
case

Ex: The Fibonacci numbers are

$$1, 1, 2, 3, 5, 8 \ldots$$

so $F_n = F_{n-1} + F_{n-2}$

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Proof by Example (be careful!)

Q: Which of the following could be proved using an example:

A: $\forall x\, P(x)$ B: $\neg \exists x\, P(x)$ $\boxed{C: \neg \forall x\, P(x)}$

$= \forall x \neg P(x)$

D: $\forall x \in \mathbb{Z}\, \neg P(x)$

need to prove for all x, and one example can't prove

To show P(x) is not true for all x, just need to find one example

Can prove statements of the form

"There exist a (number such that ...) is true"

We give an example...

"Not all (numbers satisfy ....)

We give a counter example...

ex: Prove: not all students in this class were born in the same month