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Now we can get back to proofs!

Atomic Statement: statement that cannot be broken into smaller statements

Truth Table Proofs

Prove the following statement is true:

If you eat a raw egg everyday, then you will win the lottery, or if you win the lottery you will lose your job.

P

Q

R

1. Identify atomic statements and label
2. Write statement symbollically

$$(P \to Q) \vee (Q \to R)$$

3. Create truth table

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start with atomic

next build up to full

P	Q	R	$P \to Q$	$Q \to R$	$(P \to Q) \vee (Q \to R)$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

$\underbrace{\hspace{4em}}_{\text{all possible combinations}}$

Proof: Let P be the statement . . .

" Q "

" R "

Then looking at the truth table, we see $(P \to Q) \vee (Q \to R)$ is true for any truth value of P, Q, and R.

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Truth tables are only useful when you don't have too many atomic statements $\left(\# \text{ of rows } \sim 2^{(\# \text{ of atomic statements})}\right)$

Better if can avoid

Q. Explain why $(P \to Q) \vee (Q \to R)$ is true without a truth table

- Q is true or false. If it is true, $P \to Q$ is true. If it is false, $Q \to R$ is true. Either way, at least one of $P \to Q$, $Q \to R$ is true, so the whole statement is true.

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Useful tool: Logical Equivalences

ex: $P \to Q$ is logically equivalent to $\neg P \vee Q$

P	Q	$P \to Q$	$\neg P$	$\neg P \vee Q$	$(P \to Q) \leftrightarrow (\neg P \vee Q)$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Statements R and S are logically equivalent if they have the same truth value under any assignment of truth values to their atomic parts, i.e. if $R \leftrightarrow S$.

☆ If two statements are logically equivalent, you can subsitute one for the other:

$$P \to Q \Leftrightarrow \neg P \vee Q$$

$$Q \to R \Leftrightarrow \neg Q \vee R \quad , \text{ so}$$

$$(P \to Q) \vee (Q \to R) \Leftrightarrow (\neg P \vee Q) \vee (\neg Q \vee R).$$

Now we have a bunch of OR's ($\vee$'s), so the statement is true if any substatement is true. But either Q or $\neg Q$ is true, so the whole statement is true.