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Goal: Use counting rules to solve problems

Counting

Q: Suppose you have a combination lock. It has 3 dials, each with the numbers 1, 2, ... 20. If it takes you one second to test each combination, how long will it take you to test all combinations?

A) 60 sec. B) 400 sec. C) 6,840 sec. $\boxed{\text{D) 8000 sec. } \sim 2 \text{ hours}}$

This is why passwords should be long! Otherwise can exhaustively check all options. With 5 dials → ~37 days

Probably figured out using intuition. Here is a rule to help you solve these types of problems

Product Rule: Suppose a procedure can be broken down into two tasks, and there are n_1 ways to do the first task and n_2 ways to do the second. Then there are $n_1 \times n_2$ ways to do the procedure

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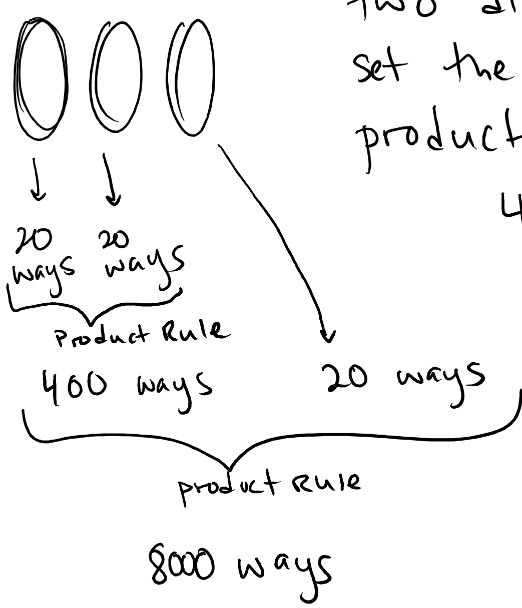
Q: Explain why answer to previous question is 8000 using product rule.

A: Let's first consider the procedure of setting the first two dials. There are 20 ways to set the first dial, and 20 ways to set the second dial. So by the product rule, there are $20 \times 20 = 40$ ways to set the first two dials. Now consider the procedure of setting all dials. There are 400 ways to set the first two dials and 20 ways to set the last dial, so by the product rule, there are $400 \times 20 = 8000$ ways to set all dials.



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SUM RULE

If you have a task that can be done in one of n_1 ways or one of n_2 ways, where the n_1 ways are all different from the n_2 ways, then there are $n_1 + n_2$ ways of completing the task.

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Combining Product & Sum Rule

Q: Suppose you and you best friend are picking into Coffrin house. There are 20 singles left and 3 doubles. If you both choose singles or both share a double, how many options of room choices do you have.

A) 23 $\boxed{\text{B) } 383}$ C) 403 D) 1200

If you both pick singles, there are 2 tasks:

1st person chooses → 20 ways

2nd person chooses → 19 ways

20 ways × 19 ways = 380 ways

There are two options: singles or doubles

singles → 380 ways

doubles → 3 ways

380 ways × 3 ways = 383

* Doesn't matter who chooses first: small example: 3 rooms:

I choose

Friend Chooses

1 — 2
1 — 3

2 — 1
2 — 3
3 — 1
3 — 2

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Let A_1, A_2 be sets

$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$$

Subtraction Rule

If you can do a task n_1 ways or n_2 ways, then the total number of ways to do a task is $n_1 + n_2$ minus the number of ways common to the two approaches.

Q: How many 5-bit strings start with 1 or end with 00?

A) 16 B) 20 C) 24 D) 32

- Start with $\underline{1}$: $2^4 = 16$
- End with 00: $2^3 = 8$
- Common $1xx00: 2^2 = 4$

$$16 + 8 - 4 = 20$$