S. KIMMEL

Goals

- Describe Iff Proof
- Describe and write Proof by cases

$P \Longleftrightarrow Q$ If and only if

$(P \to Q) \land (Q \to P)$ logically equivalent to $P \Longleftrightarrow Q$

$\Downarrow$

Structure

For the forward direction, [Proof of $P \to Q$]

For the backward direction [Proof of $Q \to P$]

S.KIMMEL

Proof By Cases

We've already seen: When you proved

$(P \to Q) \vee (Q \to R) \equiv S$ is true.

Pf

We will prove S is true for any value of Q. There are two cases: Q is true or Q is false.

P Case 1: If Q is true ..... Therefore S is true

P Case 2: If Q is false ... Therefore S is true.

ex: Prove $\forall n \in \mathbb{Z}$, $n^3 - n$ is even.

We will prove n^3-n is even for all integers n. There are two cases: n is even or n is odd.

Case 1: If n is even, $\exists k \in \mathbb{Z}: 2k = n$. Plugging in, we have

$$n^3 - n = 8k^3 - 2k = 2(4k^3 - k).$$

Since $4k^3 - k \in \mathbb{Z}$, n^3-n is even

Case 2: If n is odd, $\exists k \in \mathbb{Z}: 2k+1 = n$. Plugging in, we have

$$n^3 - n = n(n^2-1) = (2k+1)(4k+4) = 2((2k+1)(2k+2))$$

Since $(2k+1)(2k+2) \in \mathbb{Z}$, n^3-n is even.

S. KIMMEL

What if statement is not of the form $P \to Q$?

What if just have P?

Suppose you can show

P	Q	$\neg P$	$\neg Q$	$\neg P \to Q$	$\neg P \to \neg Q$
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	T	F
F	F	T	T	F	T

Proof needs to do two things

① (Direct) $\neg P \to Q$

② (Direct) $\neg P \to \neg Q$

$\therefore P$

Structure: (Prove P)

"For contradiction, assume $\neg P$"

or

"We proceed by contradiction. We assume $\neg P$."

⋮

Therefore, Q

However

⋮

Therefore, $\neg Q$, a contradiction. Thus, P.